Chiziqli algebraik tenglamalar sistemasini yechishning matritsa, Gauss va Gauss-Jordan usullari

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Annotatsiya: Chiziqli algebraaik tenglamalar tizimlarini echish usullarini ko'rib chiqiladi, unda tenglamalar soni noma'lum o'zgaruvchilar soniga teng va bitta echim mavjud. Birinchidan, biz ikkinchisiga e'tibor qaratamiz, ikkinchidan, biz tenglamalar tizimini echish usulini ko'rsatamiz, uchinchidan, Gauss usulini (noma'lum o'zgaruvchilarni izchil chiqarib tashlash usuli) tahlil qilamiz.

Kalit so'zlar: Chiziqli tenglamalar sistemasini yechishning Gauss usuli, chiziqli tenglamalar sistemasini yechishning Gauss - Jordan modifikatsiyasi, chiziqli tenglamalar sistemasining bazis yechimlari.

Matrix, Gauss and Gauss-Jordan methods for solving systems of linear algebraic equations

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Abstract: A method of solving systems of linear algebraic equations is considered, in which the number of equations is equal to the number of unknown variables and there is only one solution. First, we focus on the second, second, we show a way to solve a system of equations, and third, we analyze the Gaussian method (a method of sequential subtraction of unknown variables).

Keywords: Gaussian method of solving systems of linear equations, Gaussian-Jordan modification of systems of linear equations, basic solutions of systems of linear equations.

1. Chiziqli algebraik tenglamalar sistemasini yechishning matritsa usuli.

Ushbu n noma'lumli n ta chiziqli algebraik tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1, \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2, \\ \quad ... \quad ... \quad ... \quad ... \quad ... \quad ... \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n = b_n. \end{cases} \quad (1)$$

tenglamalar sistemada quyidagi belgilashlarni kiritamiz:

$$A = \begin{pmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{n1} & a_{n2} & ... & a_{nn} \end{pmatrix}, \ X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}, \ B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}.$$

Bu yerda, $A-$ noma'lumlar oldida turgan koeffitsiyentlardan tuzilgan matritsa; $X-$ noma'lumlardan tuzilgan matritsa; $B-$ ozod hadlardan tuzilgan matritsa.

U holda (1) tenglamalar sistemasini

$$AX = B \quad (2)$$

ko'rinishda ifodalash mumkin.

Faraz qilamiz, $\det|A| \neq 0$ bo'lsin. U holda A matritsa uchun A^{-1} teskari matritsa mavjud. $AX = B$ tenglikning har ikkala tomonini A^{-1} ga chapdan ko'paytiramiz:

$$A^{-1}AX = A^{-1}B, \ EX = A^{-1}B, \ X = A^{-1}B.$$

Hosil bo'lgan $X = A^{-1}B$ ifoda chiziqli tenglamalar sistemasini matritsalar usuli bilan yechish formulasidan iborat.

1-misol. Chiziqli tenglamalar sistemasini matritsalar usuli bilan yeching:

$$\begin{cases} 2x_1 + 2x_2 - 3x_3 = 5, \\ x_1 - x_2 + x_3 = 0, \\ 3x_1 + x_2 + x_3 = 2. \end{cases}$$

Yechish. A, X, B matritsalarni tuzib olamiz:

$$A = \begin{pmatrix} 2 & 2 & -3 \\ 1 & -1 & 1 \\ 3 & 1 & 1 \end{pmatrix}, \ X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \ B = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}.$$

Bundan, $\det|A| = -12 \neq 0.$ Teskari matritsani topamiz:

$$A_{11} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -2, \ A_{12} = -\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = 2, \ A_{13} = \begin{vmatrix} 1 & -1 \\ 3 & 1 \end{vmatrix} = 4,$$

$$A_{21} = -\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -5, \otimes \ A_{22} = \begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix} = 11, \ A_{23} = -\begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} = 4,$$

$$A_{31} = \begin{vmatrix} 2 & -3 \\ -1 & 1 \end{vmatrix} = -1, \quad A_{32} = -\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -5, \quad A_{33} = \begin{vmatrix} 2 & 2 \\ 1 & -1 \end{vmatrix} = -4,$$

$$A^{-1} = -\frac{1}{12}\begin{pmatrix} -2 & -5 & -1 \\ 2 & 11 & -5 \\ 4 & 4 & -4 \end{pmatrix}.$$

Bundan:

$$X = A^{-1}B = -\frac{1}{12}\begin{pmatrix} -2 & -5 & -1 \\ 2 & 11 & -5 \\ 4 & 4 & -4 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = -\frac{1}{12}\begin{pmatrix} -10+0-2 \\ 10+0-10 \\ 20+0-8 \end{pmatrix} = -\frac{1}{12}\begin{pmatrix} -12 \\ 0 \\ 12 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

Demak, $x_1 = 1$, $x_2 = 0$, $x_3 = -1$ yoki $X = (1;0;-1)^t$.

Agar sistema matritsasining rangi tenglama noma’lumlari sonidan kichik bo’lsa ham uning yechimini teskari matritsa usulida topish mumkin. Buni quyidagi misolda ko’rib chiqamiz.

2-misol. Ushbu chiziqli tenglamalar sistemasini teskari matritsa usulida yeching:

$$\begin{cases} x_1 - 2x_2 + 3x_3 - 5x_4 = 2, \\ 2x_1 + x_2 + 4x_3 + x_4 = -3, \\ 3x_1 - 3x_2 + 8x_3 - 2x_4 = -1, \\ 2x_1 - 2x_2 + 5x_3 - 12x_4 = 4 \end{cases}$$

Yechish. Tenglamalar sistemasi matritsasi A va kengaytirilgan matritsasi $(A|B)$

$$A = \begin{pmatrix} 1 & -2 & 3 & -5 \\ 2 & 1 & 4 & 1 \\ 3 & -3 & 8 & -2 \\ 2 & -2 & 5 & -12 \end{pmatrix}, \quad (A|B) = \left(\begin{array}{cccc|c} 1 & -2 & 3 & -5 & 2 \\ 2 & 1 & 4 & 1 & -3 \\ 3 & -3 & 8 & -2 & -1 \\ 2 & -2 & 5 & -12 & 4 \end{array}\right)$$

larning rangini topib

$$\left(\begin{array}{cccc|c} 1 & -2 & 3 & -5 & 2 \\ 2 & 1 & 4 & 1 & -3 \\ 3 & -3 & 8 & -2 & -1 \\ 2 & -2 & 5 & -12 & 4 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & -2 & 3 & -5 & 2 \\ 0 & 5 & -2 & 11 & -7 \\ 0 & 3 & -1 & 13 & -7 \\ 0 & 2 & -1 & -2 & 0 \end{array}\right) \Rightarrow$$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & -2 & 3 & -5 & 2 \\ 0 & 5 & -2 & 11 & -7 \\ 0 & 0 & 1 & 32 & -14 \\ 0 & 0 & -1 & -32 & 14 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & -2 & 3 & -5 & 2 \\ 0 & 5 & -2 & 11 & -7 \\ 0 & 0 & 1 & 32 & -14 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$r(A)=r\left(A|B\right)=3$ ekanligini ko‘ramiz. Uning minori

$$\Delta=\begin{vmatrix}1 & -2 & 3\\ 2 & 1 & 4\\ 3 & -3 & 8\end{vmatrix}=8-18-24-9+32+12=1$$

noldan farqli. Shuning uchun to‘rtinchi tenglamani tashlab yuboramiz, qolgan tenglamalarda x_4 qatnashgan hadlarni o‘ng tomonga o‘tkazamiz.

$$\begin{cases}x_1-2x_2+3x_3=2+5x_4,\\ 2x_1+x_2+4x_3=-3-x_4,\\ 3x_1-3x_2+8x_3=-1+2x_4.\end{cases}$$

Bu sistemani teskari matritsa usuli bilan yechamiz. Avval asosiy matritsa teskarisini Gauss - Jordan usulida topamiz:

$$\left(\begin{array}{ccc|ccc}1 & -2 & 3 & 1 & 0 & 0\\ 2 & 1 & 4 & 0 & 1 & 0\\ 3 & -3 & 8 & 0 & 0 & 1\end{array}\right)\Rightarrow\left(\begin{array}{ccc|ccc}1 & -2 & 3 & 1 & 0 & 0\\ 0 & 5 & -2 & -2 & 1 & 0\\ 0 & 3 & -1 & -3 & 0 & 1\end{array}\right)\Rightarrow\left(\begin{array}{ccc|ccc}1 & 7 & 0 & -8 & 0 & 3\\ 0 & -1 & 0 & 4 & 1 & -2\\ 0 & -3 & 1 & 3 & 0 & -1\end{array}\right)\Rightarrow$$

$$\Rightarrow\left(\begin{array}{ccc|ccc}1 & 0 & 0 & 20 & 7 & -11\\ 0 & 1 & 0 & -4 & -1 & 2\\ 0 & 0 & 1 & -9 & -3 & 5\end{array}\right),\quad A^{-1}=\begin{pmatrix}20 & 7 & -11\\ -4 & -1 & 2\\ -9 & -3 & 5\end{pmatrix}.$$

Tenglamalar sistemasining umumiy yechimni topish uchun $X=A^{-1}\cdot B$ amalni bajaramiz:

$$X=\begin{pmatrix}20 & 7 & -11\\ -4 & -1 & 2\\ -9 & -3 & 5\end{pmatrix}\cdot\begin{pmatrix}2+5x_4\\ -3-x_4\\ -1+2x_4\end{pmatrix}=\begin{pmatrix}10+71x_4\\ -7-15x_4\\ -14-32x_4\end{pmatrix}$$

Javob: $X=\left(30+71x_4;\,-7-15x_4;\,-14-32x_4;\,x_4\right)^t,\ \ x_4\in R$

x_4 ga ixtiyoriy qiymatlar berib $x_1,\ x_2,\ x_3$ noma’lumlarning mos qiymatlarini topamiz. Sistema cheksiz ko‘p yechimga ega.

3-misol. Quyidagi tenglamani yeching:

$$\begin{pmatrix}0 & 1\\ 2 & 1\end{pmatrix}X\begin{pmatrix}1 & 3\\ 3 & 6\end{pmatrix}=\begin{pmatrix}4 & 0\\ 5 & 8\end{pmatrix}$$

Yechish. Tenglamaga quyidagi belgilashlarni kiritamiz:

$$A=\begin{pmatrix}0 & 1\\ 2 & 1\end{pmatrix},\ B=\begin{pmatrix}1 & 3\\ 3 & 6\end{pmatrix},\ C_1=\begin{pmatrix}4 & 0\\ 5 & 8\end{pmatrix}.$$

U holda berilgan tenglama

$$A\cdot X\cdot B=C$$

ko‘rinishni oladi.

Agar AXB ifodaning chap tomondan A^{-1} va o‘ng tomondan B^{-1} ga ko‘paytirsak, hamda $A^{-1}A=E,\ EX=X$, $BB^{-1}=E$ va $XE=X$ ekanligini hisobga olsak quyidagi yechimga ega bo‘lamiz:

$$X=A^{-1}CB^{-1}=-\frac{1}{2}\begin{pmatrix}1 & -1\\ -2 & 0\end{pmatrix}\cdot\begin{pmatrix}4 & 0\\ 5 & 8\end{pmatrix}\cdot\begin{pmatrix}-2 & 1\\ 1 & -\frac{1}{3}\end{pmatrix}=$$

$$=-\frac{1}{2}\begin{pmatrix}-1 & -8\\ -8 & 0\end{pmatrix}\cdot\begin{pmatrix}-2 & 1\\ 1 & -\frac{1}{3}\end{pmatrix}=\begin{pmatrix}3 & -\frac{5}{6}\\ -8 & 4\end{pmatrix}.$$

Mashqni bajaring. Quyidagi tenglamalarni yeching:

1) $\begin{pmatrix}0 & 1\\ 3 & 1\end{pmatrix}X\begin{pmatrix}1 & 3\\ 3 & 2\end{pmatrix}=\begin{pmatrix}4 & 0\\ 2 & 8\end{pmatrix}$, 2) $\begin{pmatrix}0 & 1\\ 5 & 1\end{pmatrix}X\begin{pmatrix}2 & 3\\ 3 & 2\end{pmatrix}=\begin{pmatrix}4 & 0\\ 2 & 5\end{pmatrix}$,

3) $\begin{pmatrix}2 & 1\\ 3 & 1\end{pmatrix}X\begin{pmatrix}1 & 3\\ 2 & 5\end{pmatrix}=\begin{pmatrix}4 & 0\\ 2 & 1\end{pmatrix}$.

Agar sistemada $m\neq n$ va $r(A)\neq m$ bo‘lib, $r(A)=r(A|B)$ bo‘lgan holda ham teskari matritsa usulidan foydalanib uning yechimini topsa bo‘ladi.

2. Chiziqli algebraik tenglamalar sistemasini yechishning Gauss usuli.

n noma’lumli n ta chiziqli tenglamalar sistemasi berilgan bo‘lsin:

$$\begin{cases}a_{11}x_1+a_{12}x_2+....+a_{1n}x_n=b_1,\\ a_{21}x_1+a_{22}x_2+....+a_{2n}x_n=b_2,\\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\\ a_{n1}x_1+a_{n2}x_2+....+a_{nn}x_n=b_n.\end{cases}\quad(1)$$

n noma’lumli n ta chiziqli tenglamalar sistemasini Gauss usuli bilan yechish ikki bosqichda (dastlab chapdan o‘ngga, so‘ngra o‘ngdan chapga qarab) amalga oshiriladi.

1-bosqich. (1) sistemani uchburchak ko‘rinishga keltirishdan iborat.

Buning uchun, $a_{11}\neq 0$, deb (agar $a_{11}=0$ bo‘lsa, 1- tenglamani $a_{i1}\neq 0$ bo‘lgan i - tenglama bilan o‘rin almashtiriladi) birinchi tenglamaning chap va o‘ng tomoni a_{11} ga bo‘linadi. So‘ngra, 1 tenglama $-\frac{a_{i1}}{a_{11}}$ ga ko‘paytirilib, i-tenglamaga qo‘shiladi. Bunda, sistemaning 2-tenglamasidan boshlab x_1 noma’lum yo‘qotiladi. Bu jarayonni $n-1$ marotaba takrorlab quyidagi uchburchaksimon sistema hosil qilinadi:

$$\begin{cases} x_1 + a_{12}x_2 + a_{13}x_3 + ... + a_{1n}x_n = b_1, \\ \qquad x_2 + a_{23}x_3 + ... + a_{2n}x_n = b_2, \\ .... \\ \qquad\qquad\qquad a_{nn}x_n = b_n. \end{cases}$$

2-bosqich. Oxirgi sistemani yechishdan iborat. Bunda, dastlab sistemaning oxirgi tenglamasidan x_n topilib, undan oldingi tenglamaga qo‘yiladi va undan x_{n-1} topiladi. Shu jarayon davom ettirilib, nihoyat 1-tenglamadan x_1 topiladi.

Sistema Gauss usuli bilan yechilganda uchburchaksimon shaklga kelsa u yagona yechimga ega bo‘ladi. Agar sistema pog’onasimon shaklga kelsa u cheksiz ko‘p yechimga ega bo‘ladi yoki yechimga ega bo‘lmaydi.

Tenglamalar sistemasini yechishning Gauss usuli noma’lumlarni ketma-ket yo‘qotish usuli, deb ham ataladi. Bu jarayonni kattaroq teglamalar sistemasiga qo‘llash mumkin, chunki bu juda samarali. Quyidagi misolni ko‘rib chiqamiz:

5-misol. Tenglamalar sistemasining barcha mumkin bo‘lgan yechimlarini toping.

$$\begin{cases} 2x_2 - x_3 = -7, \\ x_1 + x_2 + 3x_3 = 2, \\ -3x_1 + 2x_2 + 2x_3 = -10. \end{cases}$$

Yechish. Dastlab ikkinchi va uchinchi tenglamadagi x_1 noma’lumni yo‘q qilinadi va keyin x_2 noma’lumni uchinchi tenglamadan yo‘qotamiz. Keyin faqat x_3 noma’lum qoladi. Lekin biz dastlabki 2 ta tenglamaning o‘rnini almashtirishdan boshlaymiz:

$$\begin{cases} x_1 + x_2 + 3x_3 = 2, \\ 2x_2 - x_3 = -7, \\ -3x_1 + 2x_2 + 2x_3 = -10 \end{cases}$$

2-tenglamada x_1 yo‘q. Keyingi qadamda 1-tenglamani ishlatib 3-tenglamadagi x_1 noma’lumni yo‘qotamiz. Bu jarayon 1-tenglamani 3 ga ko‘paytirib 3-tenglamaga qo‘shish orqali bajariladi.

$$\begin{cases} x_1 + x_2 + 3x_3 = 2, \\ 2x_2 - x_3 = -7, \\ 5x_2 + 11x_3 = -4. \end{cases}$$

Keyingi bosqichda 2-tenglamani $\frac{1}{2}$ ga ko‘paytirib, x_2 ning koeffitsiyentini 1 ga aylantiramiz.

$$\begin{cases} x_1 + x_2 + 3x_3 = 2, \\ x_2 - \frac{1}{2}x_3 = -\frac{7}{2}, \\ 5x_2 + 11x_3 = -4. \end{cases}$$

Oxirgi tenglamalar sistemasidagi 2-tenglamani -5 ga ko‘paytirib 3-tenglamaga qo‘shamiz. x_2 ni yo‘qotamiz.

$$\begin{cases} x_1 + x_2 + 3x_3 = 2, \\ x_2 - \frac{1}{2}x_3 = -\frac{7}{2}, \\ \frac{27}{2}x_3 = \frac{27}{2}. \end{cases}$$

So‘ng, oxirgi tenglamani $\frac{2}{27}$ ga ko‘paytirib $x_3 = 1$ qiymatni topamiz. Bu qiymatni ikkinchi tenglamaga qo’yib, $x_2 = -3$ qiymatni hosil qilamiz. $x_3 = 1$ va $x_2 = -3$ qiymatlarni birinchi tenglamaga qo’yib $x_1 = 2$ qiymatni olamiz. Shunday qilib, sistema yagona $(2;-3;1)$ yechimga ega.

6-misol. Chiziqli tenglamalar sistemasini Gauss usuli bilan yeching:

$$\begin{cases} 2x_1 - x_2 - 4x_3 = -1, \\ 3x_1 - 2x_2 + x_3 = -9, \\ x_1 + 4x_2 - 2x_3 = 4. \end{cases}$$

Yechish. Chiziqli tenglamalar sistemasidagi noma’lumlarni ketma-ket yo‘qotib yechimni topamiz:

$$\begin{cases} 2x_1 - x_2 - 4x_3 = -1, \\ 3x_1 - 2x_2 + x_3 = -9, \\ x_1 + 4x_2 - 2x_3 = 4 \end{cases} \Rightarrow \begin{cases} x_1 + 4x_2 - 2x_3 = 4, \\ 2x_1 - x_2 - 4x_3 = -1, \\ 3x_1 - 2x_2 + x_3 = -9 \end{cases} \Rightarrow \begin{cases} x_1 + 4x_2 - 2x_3 = 4, \\ 9x_2 = 9, \\ 14x_2 - 7x_3 = 21. \end{cases}$$

$$\Rightarrow \begin{cases} x_1 + 4x_2 - 2x_3 = 4, \\ x_3 - 2x_2 = -3, \\ x_2 = 1. \end{cases}$$

$x_2 = 1$ qiymatni ikkinchi tenglamaga qo’yib, $x_3 = -1$ qiymatni hosil qilamiz. $x_2 = 1$ va $x_3 = -1$ qiymatlarni birinchi tenglamaga qo’yib $x_1 = -2$ qiymatni olamiz. Shunday qilib, sistema yagona $(-2;1;-1)$ yechimga ega.

Tenglamalar sistemasida noma'lumlar soni tenlamalar sonidan ko'p bo'lsa ham, ya'ni sistema birgalikda bo'lib aniq bo'lmasa ham uning yechimini Gauss usulida topish mumkin. Buni quyidagi misolda ko'rib chiqamiz.

7-misol. Quyidagi chiziqli tenglamalar sistemasini Gauss usuli bilan yeching:

$$\begin{cases} x_1 - x_2 + 2x_3 - x_4 = 4, \\ x_1 + x_2 + x_3 + x_4 = 10, \\ 7x_1 + 2x_2 + 8x_3 - 6x_4 = 44, \\ 5x_1 + 2x_2 + 5x_3 - 6x_4 = 30. \end{cases}$$

Yechish. Birinchi qadamda sistemadagi birinchi tenglamani o'zgarishsiz qoldirib, qolganlaridan ketma-ket x_1 noma'lumni yo'qotamiz, ikkinchi qadamda ikkinchi tenglamani qoldirib qolganlaridan x_2 noma'lumni yo'qotamiz, uchinchi qadamda uchinchi tenglamani qoldirib qolganlaridan x_3 noma'lumni yo'qotamiz. Soddalik uchun tenglamalar sistemasi o'rniga kengaytirilgan matritsa ustida ish olib boramiz:

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 4 \\ 1 & 1 & 1 & 1 & 10 \\ 7 & 2 & 8 & -6 & 44 \\ 5 & 2 & 5 & -6 & 30 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 4 \\ 0 & 2 & -1 & 2 & 6 \\ 0 & 9 & -6 & 1 & 16 \\ 0 & 7 & -5 & 1 & 10 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 4 \\ 0 & 2 & -1 & 2 & 6 \\ 0 & 0 & 3 & 16 & 22 \\ 0 & 0 & 3 & 16 & 22 \end{array}\right)$$

Hosil bo'lgan sistemada ikkita bir hil tenglamadan bittasini qoldirib, ikkinchisini tashlab yuboramiz. Shu yerda chiziqli tenglamalar sistemasini yechishning chapdan o'ngga qarab bosqichi tugadi. Tenglamalar soni noma'lumlar sonidan kichik. Endi x_4 erkli o'zgaruvchini o'ng tomonga o'tkazamiz. So'ngra o'ngdan chapga qarab harakat yordamida sistemaning barcha yechimlari topiladi.

$$\begin{cases} x_1 - x_2 + 2x_3 - x_4 = 4, \\ \quad 2x_2 - x_3 + 2x_4 = 6, \\ \quad\quad 3x_3 + 16x_4 = 22 \end{cases} \Rightarrow \begin{cases} x_1 = 8x_4 - 34/3 \\ x_2 = -(11x_4 + 2)/3 \\ x_3 = -(16x_4 - 22)/3 \end{cases} \Rightarrow$$

Javob: $\left(8x_4 - \frac{34}{3}; -\frac{11x_4 + 2}{3}; -\frac{16x_4 - 22}{3}; x_4\right),\ x_4 \in R.$

Tenglamalar sistemasini yechishda Gauss - Jordan usuli

Tenglamalar sistemasini yechishda Gauss - Jordan usulining (Gauss usulining Jordan modifikatsiyasi) mazmun-mohiyati quyidagidan iborat: dastlabki normal ko'rinishda berilgan sistemaning kengaytirilgan $(A|B)$ matritsasi quriladi. Yuqorida keltirilgan sistemaning teng kuchliligini saqlovchi elementar almashtirishlar yordamida, kengaytirilgan matritsaning chap qismida birlik matritsa hosil qilinadi.

Bunda birlik matritsadan o‘ngda yechimlar ustuni hosil bo‘ladi. Gauss - Jordan usulini quyidagicha sxematik ifodalash mumkin:

$$\left(A|B\right)\sim\left(E|X^*\right).$$

Chiziqli tenglamalar sistemasini yechish Gauss-Jordan usuli noma’lumlarni ketma-ket yo‘qotishning Gauss strategiyasi va teskari matritsa qurishning Jordan taktikasiga asoslanadi. Teskari matritsa oshkor shaklda qurilmaydi, balki o‘ng ustunda bir yo‘la teskari matritsaning ozod hadlar ustuniga ko‘paytmasi-yechimlar ustuni quriladi.

8-misol. Quyidagi chiziqli tenglamalar sistemasini Gauss-Jordan usulida yeching:

$$\begin{cases} x_1+x_2+2x_3+3x_4=1, \\ 3x_1-x_2-x_3-2x_4=-4, \\ 2x_1+3x_2-x_3-x_4=-6, \\ x_1+2x_2+3x_3-x_4=-4. \end{cases}$$

Yechish. Chiziqli tenglamalar sistemasi koeffitsiyentlaridan kengaytirilgan matritsa tuzamiz. Tenglamalar ustida bajariladigan almashtirishlar yordamida asosiy matritsani quyidagicha birlik matritsaga keltirib javobni topamiz:

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ 3 & -1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 & -6 \\ 1 & 2 & 3 & -1 & -4 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ 0 & -4 & -7 & -11 & -7 \\ 0 & 1 & -5 & -7 & -8 \\ 0 & 1 & 1 & -4 & -5 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 1 & 2 & 3 & 1 \\ 0 & 1 & 1 & -4 & -5 \\ 0 & -1 & 5 & 7 & 8 \\ 0 & 4 & 7 & 11 & 7 \end{array}\right) \Rightarrow$$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 7 & 6 \\ 0 & 1 & 1 & -4 & -5 \\ 0 & 0 & 6 & 3 & 3 \\ 0 & 0 & 3 & 27 & 27 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 1 & 7 & 6 \\ 0 & 1 & 1 & -4 & -5 \\ 0 & 0 & 1 & 9 & 9 \\ 0 & 0 & 2 & 1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 0 & -13 & -14 \\ 0 & 0 & 1 & 9 & 9 \\ 0 & 0 & 0 & -17 & -17 \end{array}\right) \Rightarrow$$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 1 & 0 & -13 & -14 \\ 0 & 0 & 1 & 9 & 9 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$$

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