Diskret parametrli ikkinchi tartibli operatorli matritsaning muhim va diskret spektrlari

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Annotatsiya: Ushbu maqolada Fok fazosining nol zarrachali va bir zarrachali qism fazolarining to‘g‘ri yig‘indisida aniqlangan diskret parametrli ikkinchi tartibli operatorli matritsaning spektri tadqiq qilingan. Dastlab uning muhim spektri tavsiflangan. Nollari to‘plami qaralayotgan operatorning diskret spektri bilan ustma-ust tushuvchi Fredgolm determinant qurilgan. Diskret spektrning aniq ko‘rinishi topilgan.

Kalit so’zlar: operatorli matritsa, diskret parametr, spektr, muhim spektr, diskret spektr, Fredgolm determinanti.

Significant and discrete spectra of a second-order operator matrix with a discrete parameter

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Abstract: This paper examines the spectrum of a second-order operator matrix with a discrete parameter defined in the correct sum of the zero-particle and single-particle part spaces of the Fock space. Initially, its important spectrum was described. A Fredholm determinant is constructed that overlaps the discrete spectrum of the operator under consideration. A clear view of the discrete spectrum has been found.

Keywords: operator matrix, discrete parameter, spectrum, critical spectrum, discrete spectrum, Fredholm determinant.

H_0:=C orqali bir o‘lchamli kompleks fazoni, $H_1:=L_2[-\pi;\pi]$ orqali, $[-\pi;\pi]$ da aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatlarni qabul qiluvchi) funksiyalarning Gilbert fazoni belgilaymiz. H_1 va H_2 fazolarning to‘g‘ri yig‘indisini H orqali belgilaymiz, ya’ni $H:=H_0\oplus H_1$.

H fazoning f elementi $f=(f_0,f_1)$ kabi tasvirlanadi. Bu yerda $f_0\in H$ va $f_1\in H_1$. H fazoning $f=(f_0,f_1)$ va $g=(g_0,g_1)$ elementlari uchun ularning skalyar ko‘paytmasi

$$(f,g)=(f_0,g_0)_0+(f_1,g_1)_1$$

tenglik yordamida topiladi. Bunda

$$(f_0,g_0) = f_0\cdot\overline{g_0}\ ;$$

$$(f_1,g_1)_1 = \int_{-\pi}^{\pi} f_1(t)\overline{g_1(t)}dt.$$

Xuddi shuningdek $f=(f_0,f_1)\in H$ elementning normasi

$$||f||=\sqrt{||f_0||_0^2 + ||f_1||_1^2}$$

tenglik yordamida aniqlanadi. Bunda

$$||f_0||_0=|f_0|$$

$$||f||_1=\sqrt{\int_{-\pi}^{\pi} |f(t)|_1^2 dt}.$$

Odatda H Gilbert fazosiga Fok fazosining qirqilgan ikki zarrachali qism fazosi deyiladi.

Bizga yaxshi ma'lumki, H Gilbert fazosida aniqlangan har qanday chiziqli chegaralangan B operator hamisha ikkinchi tartibli operatorli matritsa ko'rinishida ya'ni

$$B=\begin{pmatrix} B_{00} & B_{01} \\ B_{10} & B_{11} \end{pmatrix}$$

ko'rinishida tasvirlanadi. Bunda $B_{ij}:H_j\rightarrow H_i$, i,j=0,1 operatorlar chiziqli chegaralangan operatorlardir.

Ta'kidlash joizki, B operator o'z-o'ziga qo'shma bo'lishi uchun

$$B_{00}^* = B_{00},\ B_{11}^*=B_{11},\ B_{01}^* = B_{10}$$

tengliklarning bajarilishi zarur va yetarlidir.

Ushbu maqolada H.Gilbert fazosida quyidagi ko'rinishdagi diskret parametrli ikkinchi tartibli operatorli matritsalarning spektral xossalari o'rganiladi:

$$A_\alpha(m) = \begin{pmatrix} A_{00}(m) & \alpha A_{01} \\ \alpha A_{01}^* & A_{11}(m) \end{pmatrix}.$$

Bunda

$$A_{ii}(m): H_i \rightarrow H_i\ ,\ i=0,1 \text{ va } A_{01}: H_1 \rightarrow H_0$$

matritsaviy elementlar quyidagi tengliklar yordamida ta'sir qiladi:

$$A_{00}(m)f_0=m\varepsilon f_0,\ f_0\in H_0;$$

$$A_{01}f_1=\int_{-\pi}^{\pi} v(t)f_1(t)dt,\ f_1\in H_1;$$

$$(A_{11}(m)f_1)(x)=(m\varepsilon+u(x))f_1(x),\ f_1\in H_1.$$

Bunda $m\in Z$; $\varepsilon > 0$; $\alpha > 0$; $v(\cdot)$ va $u(\cdot)$ funksiyalar $[-\pi;\pi]$ da aniqlangan haqiqiy qiymatli uzluksiz funksiyalar.

A_{01}^* orqali A_{01} operatorga qo'shma operator belgilangan bo'lib, bu operator

$$A_{01}^*:H_0\rightarrow H_1,\ (A_{01}^*f_0)(x)=v(x)f_0,\ f_0\in H_0$$

tenglik bilan aniqlanadi.

Haqiqatdan ham

$$(A_{01}f_1,f_0)_0=A_{01}f_1\overline{f_0}=\int_{-\pi}^{\pi} v(t)f_1(t)dt\cdot\overline{f_0}=\int_{-\pi}^{\pi} f_1(t)(v(t)\overline{f_0})dt=$$
$$=\int_{-\pi}^{\pi} f_1(t)\overline{(v(t)f_0)}dt=(f_1,A_{01}^* f_0)_1.$$

Bunda biz v(x) funksiyaning haqiqiy qiymatli funksiya ekanligidan, ya’ni

$$\overline{v(x)}=v(x),\ x\in[-\pi;\pi]$$

ekanligidan foydalandik. Yuqoridagi tenglikka ko‘ra

$$(A_{01}^*,f_0)(x)=v(x)f_0,\ f_0\in H_0$$

tenglik o‘rinli ekan.

Zamonaviy matematik fizikada A_{01} operatorga yo‘qotish operatori, A_{01}^* operatorga paydo qilish operatori, α soniga esa ta’sirlashish parametri deyiladi. Bundan tashqari, $A_\alpha(m)$ operator panjaradagi soni saqlanmaydigan va ikkitadan oshmaydigan zarrachalar sistemasiga mos matrisaviy operatorni tavsiflaydi. Zamonaviy matematik fizikada tadqiq qilinayot umumlashgan Fridrixs modeli deb ham ataladi.

Dastlab Fridrixs modeli haqida dastlabki tushunchalarni keltirib o‘tamiz.

$L_2[-1;1]$ Gilbert fazosida

$$H_\lambda f(x)=xf(x)+\lambda\int_{-1}^{1}K(x;y)f(y)dy$$

ko‘rinishida ta’sir qiluvchi H_λ operatorni qaraymiz. Bu yerda λ haqiqiy musbat son, K(x;y) esa $[-1;1]^2$ da aniqlangan haqiqiy qiymatli uzluksiz simmetrik funksiya, ya’ni K(x;y)=K(y;x) tenglik o‘rinlidir. Bu operator birinchi marta K.Fridrixs tomonidan uzluksiz spektr qo‘zg‘alishlari nazariyasi modeli sifatida [1] ishda qaralgan. Bu maqolada K(x;y) yadro o‘zining o‘zgaruvchilarining uzluksiz funksiyasi bo’lib, Gyolder shartini va

$$K(x;-1)=K(x;1)=K(-1;y)=K(1;y)=0,\ x,y\in[-1;1]$$

shartlarni qanoatlantirishi talab qilingan.

Fridrixs tomonidan $\lambda\in R$ parametrning yetarlicha kichik qiymatlarida H_λ va H_0 operatorlar unitar ekvivalent ekanligi isbotlangan, ya’ni H_λ operator $[-1;1]$ ga teng bo‘lgan sodda Lebeg spektriga ega ekanligi ko‘rsatilgan. 1948-yilda Fridrixs o’zining [2] ishida o’z modelini quyidagicha umumlashtirish masalasini taklif qilgan: birinchidan, $[-1;1]$ o‘rniga haqiqiy sonlar o‘qidagi ixtiyoriy chekli yoki cheksiz bo‘lgan ℓ intervalni qarash; ikkinchidan, qiymatlari biror abstrakt Gilbert fazosi bo‘lgan f funksiyalarni qarash. ℓ interval cheksiz bo’lgan holda K(x;y) yadro cheksizlikda kamayuvchi bo‘lsin degan qo‘shimcha shart kiritib Fridrixs bu nisbatan umumiy holda H_λ va H_0 operatorlarning unitar ekvivalent ekanligini isbotlagan.

Keyinchalik Fridrixsning [1] va [2] ishlari O.A.Ladijenskiy, D.Faddeyevlar tomonidan [3] maqolada va L.D.Faddeyev tomonidan [4] maqolada rivojlantirilgan.

Panjaradagi Fridrixs modeli bilan bog'liq tadqiqotlar [5-19] ishlarda olib borilgan. Umumlashgan Fridrixs modelining ayrim spektral xossalari [20-30] ishlarda o'rganilgan. Bu xossalar o'z navbatida panjaradagi soni saqlanmaydigan va uchtadan oshmaydigan zarrachalar sistemasiga mos 3-tartibli operatorli matrisalarning muhim va diskret spektrlarini tadqiq qilishda foydalanilgan.

Quyida biz $A_\alpha(m)$ umumlashgan Fridrixs modelining muhim spektrini aniqlaymiz. Unga mos keluvchi Fredgolm determinantini quramiz. Uning nollari to'plami sifatida $A_\alpha(m)$ operatorning diskret spektrini topamiz.

1-tasdiq. $A_\alpha(m)$*- chiziqli operator*.

Isbot: Agar f va g lar H fazoning ixtiyoriy elementlari bo'lsa, u holda

$$f=(f_0,f_1)\in H,\ f_0\in H_0,\ f_1\in H_1;$$
$$g=(g_0,g_1)\in H,\ g_0\in H_0,\ g_1\in H_1;$$

bo'lib,

$$af+bg=(af_0+bf_0,af_1+bg_1)\in H.$$

Endi

$$A_\alpha(m)(af+bg)=a\,A_\alpha(m)f+b\,A_\alpha(m)g \quad (1)$$

ekanligini tekshiramiz.

1-usul. (1)-tenglikni to'g'ridan-to'g'ri tekshiramiz:

$$A_\alpha(m)(af+bg)=\begin{pmatrix} A_{00}(m) & \alpha A_{01} \\ \alpha A_{01}^* & A_{11}(m) \end{pmatrix}$$
$$\begin{pmatrix} af_0+bg_0 \\ af_1+bg_1 \end{pmatrix}=\begin{pmatrix} A_{00}(m)(af_0+bg_0)+\alpha A_{01}(af_1+bg_1) \\ \alpha A_{01}^*(af_0+bg_0)+A_{11}(m)(af_1+bg_1) \end{pmatrix}=$$
$$=\begin{pmatrix} m\varepsilon(af_0+bg_0)+\alpha\int_{-\pi}^{\pi}v(t)(af_1(t)+bg_1(t))dt \\ \alpha v(x)(af_0+bg_0)+(m\varepsilon+u(x))(af_1(x)+bg_1(x)) \end{pmatrix}=$$
$$=\begin{pmatrix} am\varepsilon f_0+a\alpha\int_{-\pi}^{\pi}v(t)f_1(t)dt \\ a\alpha v(x)f_0+a(m\varepsilon+u(x))f_1(x) \end{pmatrix}+\begin{pmatrix} am\varepsilon g_0+a\alpha\int_{-\pi}^{\pi}v(t)g_1(t)dt \\ a\alpha v(x)g_0+a(m\varepsilon+u(x))g_1(x) \end{pmatrix}=$$
$$=a\,A_\alpha(m)f+b\,A_\alpha(m)g.$$

Demak $A_\alpha(m)$ chiziqli operator ekan.

2-usul. Quyidagi

$$A_{00}(m)(af_0+bg_0)=aA_{00}(m)f_0+bA_{00}(m)g_0;$$
$$A_{01}(af_1+bg_1)=aA_{01}f_1+bA_{01}g_1;$$
$$A_{01}^*(af_0+bg_0)=aA_{01}^*f_0+bA_{01}^*g_0;$$
$$A_{11}(m)(af_1+bg_1)=aA_{11}(m)f_1+bA_{11}(m)g_1$$

tengliklarni tekshiramiz.

1) $A_{00}(m)(af_0+bg_0)=m\varepsilon(af_0+bg_0)= m\varepsilon af_0 + m\varepsilon bg_0 = a\,A_{00}(m)f_0 + b\,A_{00}(m)g_0$;

2) $A_{01}(af_1+bg_1)=\int_{-\pi}^{\pi}v(t)(af_1(t)+bg_1(t))dt=$

$=a\int_{-\pi}^{\pi}v(t)f_1(t)dt+b\int_{-\pi}^{\pi}v(t)g_1(t)dt=aA_{01}(m)f_1+bA_{01}(m)g_1$;

3) $(A_{01}^*(af_0+bg_0))(x)=$

$\alpha v(x)(af_0+bg_0)=a\,\alpha v(x)f_0+b\alpha v(x)g_0=a(A_{01}^*f_0)(x)+b(A_{01}^*g_0)(x)$;

4) $(A_{11}(m)(af_1+bg_1))(x) = (m\varepsilon+u(x))(af_1(x)+bg_1(x)) =$
$=a(m\varepsilon+u(x))f_1(x)+b(m\varepsilon+u(x))g_1(x)=a(A_{11}(m)f_1)(x) + b(A_{11}(m)g_1(x));$

Demak $A_\alpha(m)$ operatorning to‘rtta matritsaviy elementlari chiziqli operator ekan. Shu sababli $A_\alpha(m)$ ning o‘zi ham chiziqli operator bo‘ladi.1-tasdiq to‘liq isbotlandi.

2-tasdiq. *$A_\alpha(m)$ chegaralangan operator.*

Isbot. Tasdiqni isbotlash uchun shunday $C_\alpha(m)>0$ soni topilib, barcha f∈H elementlar uchun

$$||A_\alpha(m)f|| \leq C_\alpha(m)||f|| \quad (2)$$

Haqiqatdan ham ,

$$||A_\alpha(m)f||^2=|m\varepsilon f_0 + \alpha\int_{-\pi}^{\pi} v(t)f_1(t)dt|^2+\int_{-\pi}^{\pi}|\alpha v(x)f_0 + (m\varepsilon + u(x)f_1(x)|^2dx \leq$$

$$2m^2\varepsilon^2|f_0|^2 + 2\alpha^2|\int_{-\pi}^{\pi} v(t)f_1(t)dt|^2 + 2\alpha^2\int_{-\pi}^{\pi}|v(t)|^2dt\cdot|f_0|^2 +$$

$$+2\int_{-\pi}^{\pi}|m\varepsilon + u(x)|^2\cdot|f_1(x)|^2dx \leq 2m^2\varepsilon^2|f_0|^2 + 2\alpha^2\int_{-\pi}^{\pi}|v(t)|^2dt\cdot\int_{-\pi}^{\pi}|f_1(t)|^2dt+$$

$$+2\alpha^2|\int_{-\pi}^{\pi}|v(t)|^2|f_0|^2\,dt + 2\max_{-\pi\leq x\leq\pi}|m\varepsilon + u(x)|^2\int_{-\pi}^{\pi}|f_1(x)|^2\,dx =$$

$$=(2m^2\varepsilon^2+2\alpha^2||v||^2)\cdot||f_0||_0^2 + (2\alpha^2||v||^2 + 2\max_{-\pi\leq x\leq\pi}(m\varepsilon + u(x))^2).$$

$$||f_1||_1^2 \leq C_\alpha{}^2(m)(\,||f_0||_0^2 + ||f_1||_1^2)= C_\alpha{}^2(m)||f||^2 ,$$

bu yerda,

$$C_\alpha{}^2(m)=\max\{2m^2\varepsilon^2+2\alpha^2||v||^2 ,\ 2\alpha^2||v||^2 + 2\max_{-\pi\leq x\leq\pi}(m\varepsilon + u(x))^2\}.$$

Shunday qilib

$$||A_\alpha(m)f||^2 \leq C_\alpha(m)||f|| ,$$

ya'ni (2) tengsizlik o‘rinli ekan.

Isbot jarayonida biz Koshi-Bunyakovskiy deb ataluvchi

$$|\int_{-\pi}^{\pi} v(t)f_1(t)dt|^2 \leq \int_{-\pi}^{\pi}|v(t)|^2dt\cdot\int_{-\pi}^{\pi}|f_1(t)|^2dt$$

tengsizlikdan va

$$|a+b|^2 \leq 2|a|^2 + 2|b|^2$$

elementar tengsizlikdan foydalandik.

Ta'rifga ko‘ra $A_\alpha(m)$ chegaralangan operator ekan.

3-tasdiq. *$A_\alpha(m)$ o‘z-o‘ziga qo‘shma operator.*

Isbot: Tasdiqni isbotlash uchun

$$(A_\alpha(m)f,g)=(f,\ A_\alpha(m))$$

tenglik barcha f,g ∈ H elementlar uchun bajarilishini tekshiramiz. Bu tenglikni tekshirishni ikki xil usulda amalga oshiramiz.

1-usul. To‘g‘ridan-to‘g‘ri hisoblashlar yordamida tekshirish.

2-usul esa $A_\alpha(m)$ operatorning matritsaviy elementlari uchun

$$A^*_{00}(m) = A_{00}(m);$$

$$A^*_{11}(m) = A_{11}(m);$$

tengliklarni tekshiramiz.

1-usul. Ixtiyoriy $f=(f_0,f_1)\in H$ va $g=(g_0,g_1)\in H$ elementlar uchun $(A_\alpha(m)f,g)$ ni qaraymiz:

$$(A_\alpha(m)f,g) = (A_{00}(m)f_0+\alpha A_{01}f_1g_0)_0 + (\alpha A^*_{01}f_0 + A_{11}(m)f_1,g_1)_1 =$$

$$= (m\varepsilon f_0 + \alpha\int_{-\pi}^{\pi} v(t)f_1(t)dt)\overline{g_0} + \int_{-\pi}^{\pi}(\alpha v(x)f_0 + (m\varepsilon + u(x))f_1(x))\overline{g_1(x)}dx=$$

$$= m\varepsilon f_0\overline{g_0} + \alpha\int_{-\pi}^{\pi} v(t)f_1(t)dt\overline{g_0} + \alpha\int_{-\pi}^{\pi} v(x)f_0\overline{g_1(x)}dx +$$

$$+ \int_{-\pi}^{\pi}(m\varepsilon + u(x))f_1(x)\overline{g_1(x)}dx = f_0(m\varepsilon\overline{g_0} + \alpha\int_{-\pi}^{\pi} v(t)\overline{g_1(t)}dt) +$$

$$+ \int_{-\pi}^{\pi} f_1(t)(\alpha v(t)\overline{g_0} + (m\varepsilon + u(t))\overline{g_1(t)})dt =$$

$$f_0\overline{\left(m\varepsilon g_0 + \alpha\int_{-\pi}^{\pi} v(t)g_1(t)dt\right)}+$$

$$+\int_{-\pi}^{\pi} f_1(t)\overline{(\alpha v(t)g_0 + (m\varepsilon + u(t))g_1(t))}dt = (f_0, A_{00}(m)g_0 + \alpha A_{01}g_1)_0 +$$

$$+ (f_1, \alpha A^*_{01}g_0 + A_{11}(m)g_1)_1 = (f_1,A_\alpha(m)g).$$

Demak, $A^*_{01}(m) = A_\alpha(m)$, ya'ni $A_\alpha(m)$ - o'z - o'ziga qo'shma operator ekan. Bunda biz $\varepsilon -$ haqiqiy musbat son, m - butun son, $\alpha -$ haqiqiy musbat son, v(·) va u(·) funksiyalar esa $[-\pi;\pi]$ kesmada aniqlangan haqiqiy qiymatli uzluksiz funksiya ekanligidan foydalandik.

2-usul. Ixtiyoriy $f_0,g_0 \in H_0$ elementlar uchun $(A_{00}(m)f_0, g_0)_0$ ni qaraymiz.

$$(A_{00}(m)f_0, g_0)_0 = m\varepsilon f_0\overline{g_0}=f_0m\varepsilon\overline{g_0}= f_0\overline{m\varepsilon g_0} = (f_0, A_{00}(m)g_0)_0,$$

ya'ni $A^*_{00}(m)=A_{00}(m)$ ekan.

Endi ixtiyoriy $f_1,g_1\in H_1$ elementlar uchun $(A_{11}(m)f_1,g_1)_1$ ni qaraymiz.

$$(A_{11}(m)f_1,g_1)_1 = \int_{-\pi}^{\pi}(m\varepsilon + u(t))f_1(t)\overline{g_1(t)}dt = \int_{-\pi}^{\pi} f_1(t)(m\varepsilon + u(t))\overline{g_1(t)}dt=$$

$$=\int_{-\pi}^{\pi} f_1(t)\overline{(m\varepsilon + u(t))g_1(t)}dt = (f_1,A_{11}(m)g_1)_1,$$

ya'ni $A^*_{11}(m)=A_{11}(m)$ ekan.

3-tasdiq to'liq isbotlandi.

$A_\alpha(m)$ operatorning muhim spektrini o'rganish maqsadida uni

$$A_\alpha(m)= A_0(m) + \alpha V$$

ko'rinishida tasvirlab olamiz.

Bu yerda $A_0(m)= A_\alpha(m)/_{\alpha=0}$, ya'ni

$$A_0(m)=\begin{pmatrix} A_{00}(m) & 0 \\ 0 & A_{11}(m) \end{pmatrix},$$

V-operator esa $V=\frac{1}{\alpha}(A_\alpha(m) - A_0(m))$ kabi aniqlangan, ya'ni

$$V=\begin{pmatrix} 0 & A_{01} \\ A^*_{01} & 0 \end{pmatrix}$$

V operatorning 2 o'lchamli operator ekanligi ko'rsatamiz. Dastlab ImV ni, ya'ni V operatorning qiymatlar sohasini topamiz. Aniqlanishiga ko'ra

$$\mathrm{Im}V=\{(a,bv(x)): a,b\in C\}$$

tenglik o‘rinlidir.

ImV ga tegishli bo‘lgan

$$f^{(1)} = (1,0),\ f^{(2)} = (0, v(x))$$

elementlarni tanlaymiz va ular chiziqli bog‘lanmagan elementlarni tashkil qiladi. Haqiqatdan ham

$$\beta f^{(1)} +\gamma f^{(2)} = \beta(1,0) + \gamma\big(0, v(x)\big) = (\beta, 0) + \big(0, \alpha v(x)\big) = \big(\beta, \alpha v(x)\big) = \Theta.$$

Oxirgi tenglikdan $\beta = 0$ va $\gamma v(x) = 0$, ya’ni $\beta = 0$, $\gamma = 0$ ekanligini hosil qilamiz.

Shunday qilib $f^{(1)}$ va $f^{(2)}$ elementlar chiziqli bog‘lanmagan elementlar ekan.

ImV qism fazoning ixtiyoriy f elementini olamiz, u holda shunday a,b∈ C kompleks sonlar topilib, f element f=(a,bv(x)) ko‘rinishida tasvirlanadi. Shu sababli

$$f=(a,bv(x)) = (a,0)+(b,v(x))=a(1,0)+b(0,v(x))=af^{(1)}+bf^{(2)}$$

Demak f element chiziqli bog‘lanmagan $f^{(1)}$ va $f^{(2)}$ elementlarning chiziqli kombinatsiyasidan iborat ekan. Ta’rifga ko‘ra

$$\dim \mathrm{Im}V=2,$$

ya’ni V operator 2 o‘lchamli operator ekan.

Chekli o‘lchamli qo‘zg‘alishlarda muhim spektrning o‘zgarmasligi haqidagi mashhur Veyl teoremasiga ko‘ra $A_\alpha(m)$ va $A_0(m)$ operatorlarning muhim spektrlari ustma-ust tushadi. Aniqlanishiga ko‘ra $A_0(m)$ diagonal operatordir. Shuning uchun

$$\sigma(A_0(m))=\sigma(A_{00}(m)) \cup \sigma(A_{11}(m))$$

tenglik o‘rinli bo‘ladi. $A_{00}(m)$ operatorning spektri

$$\sigma\big(A_{00}(m)\big) = \sigma_{disc}(A_{00}(m))=\{m\varepsilon\}$$

kabi aniqlanadi.

$A_{11}(m)$ operatorning spektri esa sof muhim spektr bo‘lib,

$$\sigma\big(A_{11}(m)\big) = \sigma_{ess}(A_{11}(m))=[\, m\varepsilon + u_{min}, m\varepsilon + u_{max}]$$

tenglik o‘rinlidir.

Bu yerda $A_{11}(m)$ operator $m\varepsilon + u(\cdot)$ funksiyaga ko‘paytirish operatori bo‘lganligi bois, uning spektri sof muhim spektr bo‘lib, $m\varepsilon + u(\cdot)$ funksiyaning qiymatlar sohasi bilan ustma-ust tushadi.

u_{min} va u_{max} orqali

$$u_{min}=\min_{-\pi\le x\le\pi} u(x),\ u_{max}=\max_{-\pi\le x\le\pi} u(x)$$

sonlari belgilangan.

Demak $A_\alpha(m)$ operatorning muhim spektri α ta’sirlashish parametriga bog‘liq bo‘lmasdan, u uchun

$$\sigma_{ess}(A_\alpha(m))=[\, m\varepsilon + u_{min}, m\varepsilon + u_{max}]$$

Endi $A_\alpha(m)$ operatorning diskret spektrini o‘rganinish masalasini qaraymiz.Ushbu maqsadda $\mathbb{C}\backslash[\, m\varepsilon + u_{min}, m\varepsilon + u_{max}]$ sohada regulyar bo‘lgan

$$\Delta_\alpha(m;z) = m\varepsilon - z - \alpha^2\int_{-\pi}^{\pi}\frac{\vartheta^2(t)dt}{u(t)-z}$$

funksiyani qaraymiz.

Odatda $\Delta_\alpha(m;\cdot)$ funksiyaga $A_\alpha(m)$ operatorga mos Fredgolm determinanti deyiladi. Quyidagi lemma $A_\alpha(m)$ operatorning xos qiymatlari va $\Delta_\alpha(m;\cdot)$ funksiya nollari orasidagi bog'lanishni ifodalaydi.

1-lemma. *$z\in \mathbb{C}\backslash[m\varepsilon+u_{min}, m\varepsilon+u_{max}]$ soni $A_\alpha(m)$ operatorning xos qiymati bo'lishi uchun $\Delta_\alpha(m;z)=0$ bo'lishi zarur va yetarlidir.*

Isbot. Zaruriyligi. Faraz qilaylik $z\in \mathbb{C}\backslash[\ m\varepsilon+u_{min}, m\varepsilon+u]$ operatorning xos qiymati, $f=(f_0, f_1)\in H$ esa unga mos xos vektor funksiya bo'lsin. U holda $f=(f_0, f_1)\in H$ element

$$A_\alpha(m)f = zf$$

tenglamani, ya'ni f_0 va f_1 elementlar

$$\begin{cases} m\varepsilon f_0 + \alpha\int_{-\pi}^{-\pi} v(t)f_1(t)dt = zf_0 \\ \alpha v(x)f_0 + \big(m\varepsilon + u(x)\big)f_1(x) = zf_1(x) \end{cases} \quad (3)$$

tenglamalar sistemasini qanoatlantiradi. (3) tenglamalar sistemasini unga teng kuchli bo'lgan

$$\begin{cases} (m\varepsilon - z)f_0 + \alpha\int_{-\pi}^{\pi} v(t)f_1(t)dt = 0 \\ \alpha v(x)f_0 + \big(m\varepsilon + u(x)\big)f_1(x) = zf_1(x) \end{cases} \quad (4)$$

ko'rinishida yozib olamiz. $z\in \mathbb{C}\backslash[\ m\varepsilon+u_{min}, m\varepsilon+u_{max}]$ bo'lganligi bois barcha $x\in[-\pi;\pi]$ nuqtalarda $m\varepsilon+u(x)-z\neq 0$ munosabat bajariladi. (4) tenglamalar sistemasining ikkinchi tengligidan $f_1(x)$ ni quyidagicha topib olamiz:

$$f_1(x) = \frac{-\alpha v(x)f_0}{m\varepsilon+u(x)-z} \quad (5)$$

$f_1(x)$ uchun topilgan (5) ifodani (4) tenglamalar sistemasining birinchi tengligiga qo'yamiz:

$$(m\varepsilon - z)f_0 - \alpha^2\int_{-\pi}^{\pi}\frac{\vartheta^2(t)dt}{u(t)-z}f_0 = 0$$

yoki

$$[\ m\varepsilon - z - \alpha^2\int_{-\pi}^{\pi}\frac{\vartheta^2(t)dt}{u(t)-z}\ ]f_0 = 0$$

yoki

$$\Delta_\alpha(m;z)f_0 = 0 \quad (6)$$

Agar (6) tenglikda $f_0=0$ bo'lsa, u holda (5) tenglikka ko'ra *$f_1(x)$*=0, ya'ni *$f=(0,0)=\theta$* bo'ladi. Bu esa *f* ning xos vector funksiya ekanligiga zid. Demak $f_0\neq 0$, shu sababli (6) tenglikdan $\Delta_\alpha(m;z)=0$ ekanligi kelib chiqadi. Zaruriyligi isbotlandi.

Yetarliligi. Faraz qilaylik biror $z\in \mathbb{C}\backslash[\ m\varepsilon+u_{min}, m\varepsilon+u_{max}]$ soni uchun $\Delta_\alpha(m;z)=0$ bo'lsin. $g=(g_0, g_1)$ vektor funksiya koordinatalarini

$$g_0=\text{const}\neq 0,\ g_1(x) = -\frac{\alpha v(x)g_0}{m\varepsilon+u(x)-z_0} \quad (7)$$

kabi aniqlaymiz, hamda $A_\alpha(m)g = z_0 g$ tenglik bajarilishini ko'rsatamiz:

$$A_{00}(m)g_0+\alpha A_{01}g_1 - z_0g_0 = m\varepsilon g_0+\alpha \int_{-\pi}^{\pi} v(t)(-\frac{\alpha v(t)g_0}{m\varepsilon+u(t)-z_0})dt - z_0g_0 =$$

$$= [m\varepsilon - z_0 - \alpha^2 \int_{-\pi}^{\pi} \frac{v^2(t)dt}{m\varepsilon+u(t)-z_0}]g_0 = \Delta_\alpha(m;z)g_0 = 0 \;;$$

$$\alpha(A_{01}^* g_0)(x) + (A_{11}(m)g_1)(x) - z_0 g_1(x) = \alpha v(x)g_0 + \left(m\varepsilon + u(x)g_1(x)\right) -$$

$$-z_0 g_1(x) = \alpha v(x)g_0 + (m\varepsilon + u(x) - z_0)g_1(x) = \alpha v(x)g_0 +$$

$$+(m\varepsilon + u(x) - z_0)(-\frac{\alpha v(x)g_0}{m\varepsilon+u(x)-z_0}) = (\alpha v(x) - \alpha v(x))g_0 = 0.$$

Shunday qilib qurilgan g = (g_0,g_1) vektor funksiya $A_\alpha(m)g = z_0 g$ tenglikni qanoatlantirar ekan. Aniqlanishiga ko'ra $g_0 \in H_0$ va $g_1 \in H_1$. Ta'rifga ko'ra z_0 soni $A_\alpha(m)$ operator uchun xos qiymat ekan. Lemma isbotlandi.

1-lemmadan $A_\alpha(m)$ operatorning spektri uchun quyidagi natija kelib chiqadi:

1-natija. *$A_\alpha(m)$ operatorning diskret spektri uchun*

$$\sigma_{disc}\left(A_\alpha(m)\right) = \{z \in \mathbb{C}\backslash[\, m\varepsilon + u_{min}, m\varepsilon + u_{max}]: \Delta_\alpha(m;z) = 0 \,\}$$

tenglik o'rinlidir.

1-lemma va 1-tasdiq yordamida $A_\alpha(m)$ operatorning spektri

$$\sigma\left(A_\alpha(m)\right) = [\, m\varepsilon + u_{min}, m\varepsilon + u_{max}] \cup$$

$$\cup \{z \in \mathbb{C}\backslash[\, m\varepsilon + u_{min}, m\varepsilon + u_{max}]: \Delta_\alpha(m;z) = 0 \,\}$$

kabi tavsiflanishi kelib chiqadi.

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