Sonli qatorlar (Musbat hadli qatorlarning yaqinlashish teoremalari. Leybnis teoremasi, absolyut va shartli yaqinlashish)

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Annotatsiya: Ushbu maqolada matematikaning eng qiziq mavzularidan biri bo'lgan Sonli qatorlar, (Musbat hadli qatorlarning yaqinlashish teoremalari, Leybnis teoremasi, absolyut va shartli yaqinlashish.) haqida ma'lumot berib o'tildi va mavjud muanmolarga ilmiy yondashildi. Matematik analizning ko'p masalalarini yechishda qo'shiluvchilar soni chekli yoki cheksiz bo'lgan yig'indilar bilan ish ko'rishga to'g'ri keladi. Bu cheksiz qo'shiluvchilar haqiqiy sonlardan tashqari funksiyalardan yoki vektorlardan yoki matrisalardan (yoki ma'lum bir chekli yoki cheksiz ob'ektlardan) iborat bo'lgan hollarda ularning yig'indisini topish ancha murakkab bo'ladi. Bu hollarda qo'yilgan masalalarni yechishda quyida biz o'rganadigan qatorlar nazariyasi katta ahamiyatga ega.

Kalit so'zlar: Qator haqida tushuncha, Qatorning yaqinlashishi va uzoqlashishi, Qeometrik qatorlar, Musbat hadli qatorlar, Musbat hadli qator yaqinlashishining zaruriy sharti, Garmonik qator, Dalamber, Koshining radikal va Koshining integral alomatlari, Ishoralari almashinib keluvchi qatorlar, Leybnits alomati, Absolyut va shartli yaqinlashish.

Number lines (Convergence theorems of positive series. Leibniz's theorem, absolute and conditional convergence)

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Abstract: In this article, one of the most interesting topics of mathematics, Numerical series, (Convergence theorems of positive series, Leibniz's theorem, absolute and conditional convergence) was given information and a scientific approach to existing problems was given. When solving many problems of mathematical analysis, it is necessary to work with sums with a finite or infinite number of addends. When these infinite adders consist of functions other than real numbers, or vectors or matrices (or certain finite or infinite objects), finding their sum

is more complicated. The theory of series, which we will study below, is of great importance in solving the problems posed in these cases.

Keywords: Concept of series, Convergence and divergence of series, Geometric series, Positive series, Necessary condition of convergence of positive series, Harmonic series, Dalamber, Cauchy's radical and Cauchy's integral signs, Series with alternating signs, Leibniz's sign, Absolute and conditional convergence.

Asosiy tushunchalar

Matematik analizning ko'p masalalarini yechishda qo'shiluvchilar soni chekli yoki cheksiz bo'lgan yig'indilar bilan ish ko'rishga to'g'ri keladi.

Bu cheksiz qo'shiluvchilar haqiqiy sonlardan tashqari funksiyalardan yoki vektorlardan yoki matrisalardan (yoki ma'lum bir chekli yoki cheksiz ob'ektlardan) iborat bo'lgan hollarda ularning yig'indisini topish ancha murakkab bo'ladi. Bu hollarda qo'yilgan masalalarni yechishda quyida biz o'rganadigan qatorlar nazariyasi katta ahamiyatga ega.

1-Ta'rif. Agar $a_1, a_2, a_3, ..., a_n, ...$ cheksiz haqiqiy sonlar ketma-ketligi berilgan bo'lsa, ulardan tuzilgan ushbu

$$a_1 + a_2 + a_3 + ... + a_n + ... \qquad (1)$$

ifodaga cheksiz qator (qisqacha-qator) deyiladi.

Qator qisqacha $\sum_{n=1}^{\infty} a_n$ ko'rinishda ham yoziladi.

$a_1, a_2, a_3, ..., a_n, ...$-larga qatorning hadlari deyiladi. a_n ga qatorning umumiy hadi yoki $n-$hadi deyiladi. Umumiy had yordamida qatorning ixtiyoriy hadini yozish mumkin.

Masalan, agar $a_n = \frac{1}{2^n}$ bo'lsa, u holda qator

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ... + \frac{1}{2^n} + ...$$

ko'rinishda bo'ladi.

Endi quyidagi yig'indilarni tuzaylik:

$$s_1 = a_1,\ s_2 = a_1 + a_2,\ s_3 = a_1 + a_2 + a_3, ...,\ s_n = a_1 + a_2 + a_3 + ... + a_n, ... \qquad (2)$$

yig'indilarga qatorning xususiy (yoki qismiy) yig'indilari deyiladi.

2-Ta'rif. Agar (1) qatorning $n-$xususiy yig'indisi s_n , $n \to \infty$ da $\lim_{n\to\infty} s_n = s$ chekli limitga ega bo'lsa, u holda (1) qatorga yaqinlashuvchi qator deyilib s ga esa uning yig'indisi deyiladi va s=$a_1 + a_2 + a_3 + ... + a_n + ... = \sum_{n=1}^{\infty} a_n$ ko'rinishda yoziladi.

3-Ta'rif. Agar $n \to \infty$ da (1) qatorning $n-$xususiy yig'indisi s_n ning limiti cheksiz bo'lsa yoki mavjud bo'lmasa, u holda (1) qator uzoqlashuvchi deyiladi.

Cheksiz qatorga misol sifatida kelajakda ko'p foydalaniladigan va o'rta maktab dasturidan ma'lum bo'lgan geometrik progressiyani ko'rib o'taylik.

$$a+aq+aq^2+...+aq^{n-1}+... \quad (3)$$

a geometrik progressiyaning (geometrik qatorning) birinchi hadi, $a\cdot q^{n-1}$ n-hadi, q esa mahraji bo'lib, birinchi n ta hadining yig'indisi $(|q|\neq 1)$ bo'lganda

$$s_n=a+aq+aq^2+...+aq^{n-1}=\frac{a(1-q^n)}{1-q}$$ bo'ladi.

1. $|q|<1$ bo'lsa $n\to\infty$ da $q^n\to 0$ bo'lib

$$\lim_{n\to\infty} s_n=\lim_{n\to\infty}\left(\frac{a}{1-q}-\frac{aq^n}{1-q}\right)=\frac{a}{1-q}$$ bo'ladi.

Demak (3) qator yaqinlashuvchi bo'lib yig'indisi $s=\frac{a}{1-q}$ bo'ladi.

2. $|q|>1$ bo'lsa $n\to\infty$ da $q^n\to\infty$ bo'lib, (3) qator uzoqlashuvchi bo'ladi.

3. $q=1$ bo'lsa, (3) qator $a+a+a+...+a+...$ ko'rinishda bo'lib

$s_n=a+a+a+...+a=na$ bo'ladi.

$$\lim_{n\to\infty} s_n=\lim_{n\to\infty}(an)=a\lim_{n\to\infty} n=\infty \quad (a\neq 0).$$

Demak, qator uzoqlashuvchi.

4. $q=-1, a\neq 0$ bo'lsa, (3) qator $a-a+a-a+...$ ko'rinishda bo'lib,

n juft son bo'lganda s_n=0 va n toq son bo'lganda $s_n=a$ bo'ladi. Demak, $\lim_{n\to\infty} s_n$ mavjud emas va qator uzoqlashadi.

Shunday qilib geometrik progressiya ya'ni (3) qator faqat $|q|<1$ bo'lganda yaqinlashuvchi bo'lib, $|q|\geq 1$ bo'lganda uzoqlashuvchi bo'lar ekan.

Sonli qatorlarning ba'zi xossalari

$$a_1+a_2+a_3+...+a_m+a_{m+1}+...+a_n+... \quad (1)$$

Qatorning birinchi chekli m ta hadini tashlab yuborsak, natijada

$$a_{m+1}+a_{m+2}+...+a_n+... \quad (4)$$

qator hosil bo'ladi.

1-teorema. Agar (1) qator yaqinlashuvchi (uzoqlashuvchi) bo'lsa, uning istalgan chekli m sondagi hadlarini tashlab yuborishdan hosil bo'lgan (4) qator ham yaqinlashuvchi (uzoqlashuvchi) bo'ladi va aksincha (4) qator yaqinlashuvchi (uzoqlashuvchi) bo'lsa, u holda (1) qator ham yaqinlashuvchi (uzoqlashuvchi) bo'ladi.

Isbot. (1) qatorning xususiy yig'indisi

$$s_n=(a_1+a_2+a_3+...+a_m)+(a_{m+1}+a_{m+2}...+a_n)=s_m+s_{n-m}$$

(4) qatorning xususiy yig'indisi

$$s_{n-m} = a_{m+1} + a_{m+2} ... + a_n$$

bo'lgani uchun $s_n = s_m + s_{n-m}$ dan ko'rinadiki:

a) Agar $\lim_{n\to\infty} s_n$ mavjud bo'lsa, $\lim_{n\to\infty} s_{n-m}$ ham mavjud bo'ladi, bu esa (1) qator yaqinlashuvchi bo'lsa, (4) qatorning ham yaqinlashuvchi ekanini ko'rsatadi s_m-chekli son n ga bog'liq emas).

b) Agar $\lim_{n\to\infty} s_n$ mavjud bo'lmasa yoki cheksiz bo'lsa $\lim_{n\to\infty} s_{n-m}$ ham mavjud emas yoki cheksiz bo'ladi. Bu esa (1) qator uzoqlashuvchi bo'lsa, (4) qator ham uzoqlashuvchi ekanini ko'rsatadi.

Teoremaning ikkinchi qismi ham xuddi shuningdek isbotlanadi.

2-teorema. Qator hadlariga chekli sondagi hadlar qo'shganda ham o'rinli bo'ladi.

3-Teorema. Agar

$$\boldsymbol{a_1 + a_2 + a_3 + ... + a_n + ...} \qquad \textbf{(1)}$$

qator yaqinlashuvchi bo'lib, yig'indisi s bo'lsa, u holda

$$\lambda a_1 + \lambda a_2 + \lambda a_3 + ... + \lambda a_n + ... \qquad (5)$$

qator ham yaqinlashuvchi bo'lib yig'indisi λs bo'ladi (λ -ixtiyoriy o'zgarmas).

Isboti. (1) Qator yaqinlashuvchi bo'lgani uchun

$$\lim_{n\to\infty}(a_1 + a_2 + a_3 + ... + a_n) = \lim_{n\to\infty} s_n = s$$

bo'ladi. (5) Qatorning $n-$xususiy yig'indisi

$$\lambda a_1 + \lambda a_2 + \lambda a_3 + ... + \lambda a_n$$

bo'lib, limiti esa

$$\lim_{n\to\infty}(\lambda a_1 + \lambda a_2 + \lambda a_3 + ... + \lambda a_n) = \lambda \lim_{n\to\infty}(a_1 + a_2 + a_3 + ... + a_n) = \lambda \lim_{n\to\infty} s_n = \lambda s$$

bundan (5) qatorning yaqinlashuvchi ekanligi kelib chiqadi.

4-teorema. Agar $a_1 + a_2 + a_3 + ... + a_n + ...$ va $b_1 + b_2 + b_3 + ... + b_n + ...$

qatorlar yaqinlashuvchi bo'lib, yig'indilari mos ravishda s' ва s'' bo'lsa

$$(a_1 + b_1) + (a_2 + b_2) + (a_3 + b_3) + ... + (a_n + b_n) + ... \qquad (6)$$

qator ham yaqinlashuvchi bo'ladi va uning yig'indisi $s = s' + s''$ bo'ladi.

Isbot. Shartga ko'ra

$$\lim_{n\to\infty}(a_1 + a_2 + a_3 + ... + a_n) = \lim_{n\to\infty} s'_n = s'$$

va

$$\lim_{n\to\infty}(b_1 + b_2 + b_3 + ... + b_n) = \lim_{n\to\infty} s''_n = s''$$

tengliklar o'rinli bo'ladi.

(6) Qatorning $n-$xususiy yig'indisini s_n desak

$$s_n = (a_1 + b_1) + (a_2 + b_2) + (a_3 + b_3) + ... + (a_n + b_n)$$

bo'lib,

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} (a_1 + b_1 + a_2 + b_2 + \ldots + a_n + b_n) =$$
$$= \lim_{n\to\infty}(a_1 + a_2 + a_3 + \ldots + a_n) + \lim_{n\to\infty}(b_1 + b_2 + b_3 + \ldots + b_n) = s' + s'' = s$$

Bu esa bundan (6) qatorning yaqinlashuvchi ekanligini ko'rsatadi.

Qator yaqinlashishining zaruriy sharti.

Qatorlar nazariyasining asosiy masalasi ularning yaqinlashuvchi yoki uzoqlashuvchi ekanligini ko'rsatish.

5-Teorema. Agar $a_1 + a_2 + a_3 + \ldots + a_n + \ldots$ qator yaqinlashuvchi bo'lsa, uning n-hadi n cheksizlikka intilganda nolga intiladi ya'ni

$$\lim_{n\to\infty} a_n = 0$$

Isboti. Teoremaning shartiga ko'ra

$$a_1 + a_2 + a_3 + \ldots + a_n + \ldots$$

qator yaqinlashuvchi bo'lsa,

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty}(a_1 + a_2 + a_3 + \ldots + a_n) = s$$

bo'ladi. Bu holda

$$\lim_{n\to\infty} s_{n-1} = \lim_{n\to\infty}(a_1 + a_2 + a_3 + \ldots + a_{n-1}) = s$$

ekanligi qatorning birinchi xossasiga ko'ra ravshan.

$$\lim_{n\to\infty}(s_n - s_{n-1}) = \lim_{n\to\infty} s_n - \lim_{n\to\infty} s_{n-1} = s - s = 0$$

Ikkinchi tomondan $s_n - s_{n-1} = a_n$ bo'lgani uchun

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}(s_n - s_{n-1}) = 0$$

Eslatma. Agar qator yaqinlashuvchi bo'lsa, albatta $n \to \infty$ da uning n-hadi nolga intiladi ya'ni $a_n \to 0$ bo'ladi. Agar $n \to \infty$ da qatorning n-hadi nolga intilmasa qator albatta uzoqlashuvchi bo'ladi. Agar $n \to \infty$ da qatorning n-hadi nolga intilsa ya'ni $\lim_{n\to\infty} a_n = 0$ bo'lsa, bu qatorning yaqinlashishining muqarrarligi kelib chiqmaydi. Boshqacha aytganda $\lim_{n\to\infty} a_n = 0$ dan qatorning albatta yaqinlashuvchi bo'lishi kelib chiqmaydi u qator uzoqlashuvchi bo'lishi ham mumkin.

Masalan. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots + \frac{1}{n} + \ldots$ garmonik qator deb ataluvchi qatorning $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} = 0$ bo'lgani bilan bu qator uzoqlashuvchi. Buning uzoqlashuvchi ekanligini keyinroq Koshining integral alomati yordamida isbotlanadi.

Musbat hadli qatorlar.

Agar $a_1 + a_2 + a_3 + \ldots + a_n + \ldots$ qatorning hamma hadlari manfiy bo'lmagan sonlardan iborat bo'lsa, bunday qatorga musbat hadli qator deyiladi.

$a_n > 0$ $(n = 1,2,...)$ bo'lgani uchun qatorning barcha xususiy yig'indilari monoton o'suvchi bo'lib $s_1 < s_2 < ... < s_n < ...$ bo'ladi.

Biz bilamizki monoton o'suvchi ketma-ketliklar yuqoridan chegaralangan bo'lsa uning limiti mavjud bo'lib ketma-ketlik yaqinlashuvchi bo'ladi. Demak, bu holda qator yaqinlashuvchi bo'ladi.

Agar monoton o'suvchi $s_1, s_2, ..., s_n, ...$ xususiy yig'indilar yuqoridan chegaralanmagan bo'lsa, u chekli limitga ega bo'lmaydi. Demak, bu holda qator uzoqlashuvchi bo'ladi.

6-Teorema. Musbat hadli qatorlarning yaqinlashuvchi bo'lishi uchun ularning barcha xususiy yig'indilari yuqoridan chegaralangan bo'lishi zarur va kifoya.

Musbat hadli qatorlarning yaqinlashishining etarli shartlarini ko'rib o'taylik.

Birinchi taqqoslash alomati.

$$a_1 + a_2 + a_3 + ... + {}_n + ..a \quad (1)$$

$$b_1 + b_2 + b_3 + ... + b_n + ... \quad (7)$$

musbat hadli qatorlar berilgan bo'lib biror $n > N$ nomerdan boshlab

$$a_n \le b_n \quad (*)$$

tengsizlik bajariladigan bo'lsa , (7) qatorning yaqinlashuvchi bo'lishligidan (1) qatorning yaqinlashuvchiligi yoki (1) qatorning uzoqlashuvchi bo'lishligidan (7) qatorning ham uzoqlashuvchi bo'lishligi kelib chiqadi.

Isbot. $s_n = \sum_{k=1}^{n} a_k$, $s_n' = \sum_{k=1}^{n} b_k$ bo'lsin. Ikkinchi qator yaqinlashuvchi bo'lgani uchun $\lim_{n\to\infty} s_n' = s$ bo'ladi. Teoremaning shartiga ko'ra (1) va (7) musbat hadli qatorlar bo'lgani uchun $s_n \le s_n' \le s$. Bundan (7) qatorning xususiy yig'indilari chegaralanganligi va uning yaqinlashuvchiligi keliib chiqadi.

Endi (1) qator uzoqlashuvchi bo'lsin, ya'ni $\lim_{n\to\infty} s_n = \infty.$ $(*)$ tengsizlikka ko'ra $s_n \le s_n'$. Demak, $\lim_{n\to\infty} s_n = \infty$ va qator uzoqlashuvchi.

1-*misol.* $\frac{2}{3} + \frac{1}{2}\left(\frac{2}{3}\right)^2 + \frac{1}{3}\left(\frac{2}{3}\right)^3 + ... + \frac{1}{n}\left(\frac{2}{3}\right)^n + ...$

va

$\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + ... + \left(\frac{2}{3}\right)^n + ...$ qatorlar berilgan bo'lsin.

Ravshanki

$$a_n = \frac{1}{n}\left(\frac{2}{3}\right)^n \le \left(\frac{2}{3}\right)^n = b_n$$

$\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ qator yaqinlashuvchi, demak 1-teoremaga ko'ra birinchi qator ham yaqinlashuvchi bo'ladi.

2- *misol.* $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+...+\frac{1}{\sqrt{n}}+...$ qator uzoqlashuvchi, chunki uning hadlari, ikkinchi hadidan boshlab $1+\frac{1}{2}+\frac{2}{3}+\frac{1}{4}+...+\frac{1}{n}+...$ garmonik qatorning mos hadlaridan katta, garmonik qator esa uzoqlashuvchidir.

Ikkinchi taqqoslash alomati

Agar $\lim_{n\to\infty}\frac{a_n}{b_n}=k<\infty$ limit mavjud bo'lsa, u holda (1) va (7) qatorlar bir vaqtda yaqinlashadi yoki uzoqlashadi.

3-*misol.* $\sin 1+\sin\frac{1}{2}+...+\sin\frac{1}{n}+...$ qatorni $1+\frac{1}{2}+\frac{2}{3}+\frac{1}{4}+...+\frac{1}{n}+...$ qator bilan taqqoslaymiz.

$\frac{a_n}{b_n}=\frac{\sin\frac{1}{n}}{\frac{1}{n}}$ nisbatni ko'ramiz. Ma'lumki, $\lim_{n\to\infty}=\frac{\sin\frac{1}{n}}{\frac{1}{n}}=1.$ Demak, berilgan qator uzoqlashuvchi.

4-*misol.* $\sin\frac{1}{2}+\sin\frac{1}{2^2}+...+\sin\frac{1}{2^n}+...$ qator $\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}...+\sin\frac{1}{2^n}+...$ qator bilan taqqoslaymiz. Berilgan ikkinchi qator yaqinlashuvchi, chunki $q=\frac{1}{2}$ bo'lgan cheksiz kamayuvchi geometrik progressiyadir.

$\frac{a_n}{b_n}=\frac{\sin\frac{1}{2^n}}{\frac{1}{2^n}}$ va $\lim_{n\to\infty}=\frac{\sin\frac{1}{2^n}}{\frac{1}{2^n}}=1.$ Shunday qilib qator yaqinlashuvchi.

7-*Teorema.* (Dalamber alomati). Agar

$$a_1+a_2+a_3+...+a_n+... \quad (1)$$

qatorning $(n+1)$-hadining n-hadiga nisbatan $n\to\infty$ da chekli limitga ega bo'lsa, ya'ni $\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=l$ (8)

bo'lsa, u holda

1) $l<1$ da qator yaqinlashadi;

2) $l>1$ da qator uzoqlashadi.

Isbot. 1) $l<1$. U holda l bilan 1 orasidan biror q sonni olaylik $l<q<1$ bo'lsin, (8) munosabatidan ko'rinadiki n ning biror $n=N$ nomeridan boshlab $\frac{a_{n+1}}{a_n}<\frac{a_{N+1}}{a_N}<q$ bo'ladi.

Oxirgi tezlikdan

$$\left.\begin{array}{l} a_{N+1}<a_N q \\ a_{N+2}<a_{N+1}q<a_N q^2 \\ a_{N+3}<a_{N+2}q<a_N q^3 \\ \dots\dots\dots\dots\dots\dots \\ \dots\dots\dots\dots\dots\dots \end{array}\right\} \quad (9)$$

(9) dan ko'rinadiki

$$a_1+a_2+a_3+\ldots+a_{N+1}\ldots+a_n+\ldots=\sum_{n=1}^{\infty}a_n$$

qatorning a_{N+1} dan boshlab har bir hadi $q<1$ bo'lganda yaqinlashuvchi bo'lgan

$$a_N q+a_N q^2\ldots+a_N q^n\ldots=\sum_{n=1}^{\infty}a_N q^n \quad (10)$$

qatorning tegishli hadlaridan kichik bo'ladi. Demak, taqqoslash teoremasiga ko'ra (1) qator yaqinlashuvichi bo'ladi.

2) $l>1$ bo'lsin, u holda n ning biror $n\geq N$ nomeridan boshlab

$$\frac{a_{n+1}}{a_n}>q>1\Rightarrow a_{n+1}>qa_n>a_n$$

bundan ko'rinadiki, qator yaqinlashishining zaruriy sharti

$$\lim_{n\to\infty}a_n=0$$

bajarilmaydi. Demak, (1) qator uzoqlashuvchi bo'ladi

8-Teorema. (Koshi alomati). Agar musbat hadli

$$a_1+a_2+a_3+\ldots+a_n+\ldots$$

qator uchun

$$\lim_{n\to\infty}\sqrt[n]{a_n}=l$$

chekli limit mavjud bo'lib

1) $l<1$ bo'lsa qator yaqinlashadi;

2) $l>1$ bo'lsa qator uzoqlashadi.

Teorema Dalamber alomati kabi isbotlanadi.

1-Misol. Qatorni yaqinlashuvchiligini tekshiring:

$$\frac{2}{1^2}+\frac{2^2}{2^2}+\frac{2^3}{3^2}+\ldots+\frac{2^n}{n^2}+\ldots$$

Yechish. Ma'lumki , $a_n = \frac{2^n}{n^2}$, $a_{n+1} = \frac{2^{n+1}}{(n+1)^2}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{2^{n+1}}{(n+1)^2}}{\frac{2^n}{n^2}} = 2\lim_{n\to\infty} \frac{n^2}{(n+1)^2} = 2 > 1$$

Demak, qator uzoqlashuvchi.

2-Misol. Berilgan qatorni yaqinlashuvchiligini tekshiring:

$$\frac{1}{\sqrt{2}} + \frac{3}{\left(\sqrt{2}\right)^2} + \frac{5}{\left(\sqrt{2}\right)^3} + \ldots + \frac{2n-1}{\left(\sqrt{2}\right)^n} + \ldots$$

Yechish. $a_n = \frac{2n-1}{\left(\sqrt{2}\right)^n}$, $a_{n+1} = \frac{2n+1}{\left(\sqrt{2}\right)^{n+1}}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{2n+1}{\left(\sqrt{2}\right)^{n+1}}}{\frac{2n-1}{\left(\sqrt{2}\right)^n}} = \frac{1}{\sqrt{2}} \lim_{n\to\infty} \frac{2n+1}{2n-1} = \frac{1}{\sqrt{2}} < 1$$

Demak, qator yaqinlashuvchi.

3-Misol. Qatorni yaqinlashuvchiligini ko'rsating:

$$1 + \frac{1}{\sqrt[3]{2}} + \frac{1}{\sqrt[3]{3}} + \frac{1}{\sqrt[3]{4}} + \ldots + \frac{1}{\sqrt[3]{n}} + \ldots$$

Yechish. $a_n = \frac{1}{\sqrt[3]{n}}$, $a_{n+1} = \frac{1}{\sqrt[3]{n+1}}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty n} \frac{\frac{1}{\sqrt[3]{n+1}}}{\frac{1}{\sqrt[3]{n}}} = \lim_{n\to\infty n} \sqrt[3]{\frac{n}{n+1}} = 1$$

Qatorning yaqinlashishi to'g'risida Dalamber alomati asosida xulosa chiqarish mumkin emas. Taqqoslash alomatiga ko'ra, qatorning uzoqlashuvchanligini ko'rish mumkin.

4-Misol. Berilgan qatorni yaqinlashuvchiligini tekshiring:

$$\frac{1}{\ln 2} + \frac{1}{\ln^2 3} + \ldots + \frac{1}{\ln^n (n+1)} + \ldots$$

Yechish. $\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[n]{\frac{1}{\ln^n (n+1)}} = \frac{1}{\ln(n+1)} = 0 < 1$

Demak, qator yaqinlashuvchi.

5-Misol. $\frac{2}{1}+\left(\frac{3}{2}\right)^4+\left(\frac{4}{3}\right)^9+\ldots+\left(\frac{n+1}{n}\right)^{n^2}+\ldots$ qatorni yaqinlashuvchiligini tekshiring.

Yechish. $\lim\limits_{n\to\infty}\sqrt[n]{a_n}=\lim\limits_{n\to\infty}\sqrt[n]{\left(\frac{n+1}{n}\right)^{n^2}}=\lim\limits_{n\to\infty}\left(\frac{n+1}{n}\right)^n=e>1$

qator uzoqlashuvchi.

9-Teorema. (Koshining integral alomati). Bizga hadlari o'smaydigan

$$(a_n \ge a_{n+1})$$

$$a_1+a_2+a_3+\ldots+a_n+\ldots \quad (1)$$

musbat hadli qator va uzluksiz o'smaydigan ($x\to\infty$ да $f(x)\to 0$) monoton kamayuvchi $f(x)$ funksiya berilgan bo'lib

$$f(1)=a_1, f(2)=a_2,\ldots, f(n)=a_n,\ldots$$

bo'lsa, u holda (1) qatorning yoki

$$\sum_{n=1}^{\infty} f(n) \quad (11)$$

qatorning yaqinlashuvchi bo'lishi uchun

$$\int_1^{\infty} f(x)dx$$

xosmas integralning yaqinlashuvchi bo'lishi zarur va kifoya.

Isbot. $\int_1^{\infty} f(x)dx$ xosmas integral yaqinlashuvchi degan so'z $n\to\infty$ da

$$\int_1^{n} f(x)dx \quad (12)$$

integralning limiti mavjud degan so'z.

$n\to\infty$ da (12) integralning limiti mavjud degan so'z, o'z navbatida

$$\int_1^{2} f(x)dx+\int_2^{3} f(x)dx+\ldots+\int_n^{n+1} f(x)dx+\ldots \quad (13)$$

Qatorning yaqinlashishini bildiradi, chunki (12) integral (13) qator uchun n-hususiy yig'indi ekanligini ko'rish qiyin emas ((13) ning birinchi n-1 ta o'adlarini qo'shib chiqsak (12) kelib chiqadi). Shunday qilib (1) (yoki (11)) va (13) qatorlarning bir paytda yaqinlashuvchi yoki uzoqlashuvchi ekanligini ko'rsatishimiz kerak.

$f(x)$ funksiya o'smaydigan monoton kamayuvchi bo'lgani uchun har qanday $[n,n+1]$ kesmada

$f(n+1) \leq \ f(x) \leq f(n) \ \ (n=\overline{1,\infty})$ tengsizlik o'rinli.

Bu tengsizlikni $[n,n+1]$ da integrallasak

$$\int_n^{n+1} f(n+1)dx \leq \int_n^{n+1} f(x)dx \leq \int_n^{n+1} f(n)dx \quad \Rightarrow \ \boldsymbol{f(n+1) \leq} \int_n^{n+1} f(x)dx \leq f(n) \tag{14}$$

Agar (11) qator yaqinlashuvchi bo'lsa

$$\int_n^{n+1} f(x)dx \leq f(n)$$

dan ko'rinadiki taqqoslash teoremasiga ko'ra (13) qator yaqinlashadi.

Agar (13) qator yaqinlashuvchi bo'lsa

$$f(n+1) \leq \int_n^{n+1} f(x)dx$$

dan ko'rinadiki taqqoslash teoremasiga ko'ra (11) yaqinlashuvchi bo'ladi.

Misol. Umumlashgan garmonik qator deb ataluvchi

$$1+\frac{1}{2^p}+\frac{1}{3^p}+\frac{1}{4^p}+\ldots+\frac{1}{n^p}+\ldots$$

qatorni yaqinlashuvchanlikka tekshiring.

Yechish. $a_1 = f(1) = 1, a_2 = f(2) = \frac{1}{2^p}, \ldots, a_n = f(n) = \frac{1}{n^p}, \ldots$ va $f(x) = \frac{1}{x^p}$ ekanligi ravshan, bu erda r-haqiqiy son.

Ushbu

$$\int_1^{\infty} \frac{1}{x^p}dx = \left.\frac{x^{-p+1}}{-p+1}\right|_1^{\infty} = \frac{1}{1-p}\lim_{n\to\infty} x^{-p+1}\Big|_1^{\infty} = \frac{1}{1-p}\lim_{n\to\infty}(n^{1-p}-1) \quad (\mathrm{p} \neq 1)$$

xosmas integralni hisoblaymiz.

Agar r>1 bo'lsa, u holda $\lim\limits_{n\to\infty} n^{1-p} = 0$ va $\int_1^{\infty}\frac{1}{x^p}dx = \frac{1}{1-p}$ yaqinlashuvchi;

Agar r<1 bo'lsa, u holda $\lim\limits_{n\to\infty} n^{1-p} = \infty$ va $\int_1^{\infty}\frac{1}{x^p}dx$ uzoqlashuvchi;

Agar r=1 bo'lsa, u holda $\int_1^{\infty}\frac{1}{x}dx = \ln x\Big|_1^{\infty} = \infty$ uzoqlashuvchi.

Shu sababli umumlashgan garmonik qator

r>1 bo'lsa yaqinlashuvchi,

r<1 bo'lsa uzoqlashuvchi va

r=1 bo'lsa uzoqlashuvchi bo'ladi.

Ishoralari almashinib keluvchi qatorlar. Leybnits alomati.

$$\boldsymbol{u_1 - u_2 + u_3 - u_4 + \ldots + (-1)^{n-1}u_n + \ldots = \sum_{n=1}^{\infty}(-1)^{n-1}u_n} \tag{15}$$

ko'rinishdagi qatorga ishoralari navbat bilan almashib keladigan qatorlar deyiladi.

Bu yerda $u_1, u_2, u_3, \ldots, u_n, \ldots$ musbat sonlar.

10-Teorema (Leybnis teoremasi). Agar ishorasi almashinib keluvchi

$$u_1 - u_2 + u_3 - u_4 + \ldots + (-1)^{n-1} u_n + \ldots$$

qatorda

1) Qator hadlarining absolyut qiymatlari kamayuvchi, ya'ni

$$u_1 > u_2 > u_3 > u_4 > \ldots > u_n > \ldots \quad (16)$$

bo'lsa,

2) Qator umumiy hadi u_n $n \to \infty$ da nolga intilsa:

$$\lim_{n\to\infty} u_n = 0 \quad (17)$$

u holda (15) qator yaqinlashuvchi bo'ladi.

Isbot. $n = 2m$, ya'ni juft bo'lsin

$\boldsymbol{S_{2m} = (u_1 - u_2) + (u_3 - u_4) + \ldots + (u_{2m-1} - u_{2m})}$, demak $S_{2m} > 0$ va xususiy yig'indilar ketma-ketligi S_{2m} o'suvchi. (15) shartga ko'ra har bir qavs ichidagi ifoda musbat ekanligi kelib chiqadi.

Endi S_{2m} xususiy yig'indilarni quyidagi ko'rinishda yozamiz:

$$S_{2m} = u_1 - (u_2 - u_3) - \ldots - (u_{2m-2} - u_{2m-1}) - u_{2m}$$

Bu ifodaning har bir qavs ichidagi ishoralari musbat. Shu sababli $u_1 > S_{2m}$.

Shunday qilib, S_{2m} xususiy yig'indilar ketma-ketligi o'suvchi va yuqoridan chegaralangan. Demak $\lim_{m\to\infty} S_{2m} = S$ shu bilan birgalikda $u_1 > S > 0$.

Endi toq indeksli S_{2m+1} xususiy yig'indilar ham S limitga intiladi.

Haqiqatan, ham

$$S_{2m+1} = S_{2m} +$$

bo'lgani uchun $m \to \infty$ da

$$\lim_{m\to\infty} S_{2m+1} = \lim_{m\to\infty} S_{2m} + \lim_{m\to\infty} u_{2m+1} = \lim_{m\to\infty} S_{2m} = S$$

ga ega bo'lamiz, bunda (17) shartga ko'ra

$$\lim_{m\to\infty} u_{2m+1} = 0$$

Demak, $\lim_{n\to\infty} S_n = S$, qator yaqinlashuvchi.

Misol. $\frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} - \ldots + (-1)^{n+1}\frac{1}{(n+1)^2} + \ldots$ qatorning yaqinlashuvchanligini tekshiring.

Yechish. $\frac{1}{2^2} > \frac{1}{3^2} > \frac{1}{4^2} > \ldots > \frac{1}{(n+1)^2} > \ldots$ va $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{(n+1)^2} = 0$.

Demak, qator yaqinlashuvchi.

Endi ixtiyoriy ishorali qatorlarni ko'raylik. O'zgaruvchan ishorali qatorning absolyut va shartli yaqinlashishikabi muhim tushunchalarni ko'raylik.

Absolyut va shartli yaqinlashish.

$$u_1 + u_2 + u_3 + u_4 + \ldots + u_n + \ldots \quad (18)$$

Qatorning cheksiz ko'p musbat va cheksiz ko'p manfiy hadlari bo'lsa, u holda bu qatorga o'zgaruvchan ishorali qator yoki ixtiyoriy hadli qator deyiladi.

(1) qator hadlarining absolyut qiymatlaridan tuzilgan

$$|u_1| + |u_2| + |u_3| + |u_4| + \ldots + |u_n| + \ldots \quad (19)$$

qatorni tuzaylik.

11-Teorema. Agar (19) qator yaqinlashuvchi bo'lsa, (18) qator ham yaqinlashuvchi bo'ladi.

Isbot. S_n va S_n' mos ravishda (18) va (19) qatorlarning n-xususiy yig'indilari bo'lsin. S_n^+ bilan barcha musbat va S_n^- bilan S_n xususiy yig'indidagi barcha manfiy ishorali hadlar qiymatlari yig'indisini belgilaymiz. U holda

$$S_n = S_n^+ - S_n^-, \quad S_n' = S_n^+ + S_n^-$$

Shartga ko'ra, (19) qator yaqinlashuvchi, shu sababli S_n' yig'indi S yig'indiga ega.

S_n^+ va S_n^- lar esa musbat va o'suvchi, shu bilan birgalikda $S_n^+ \le S_n' < S$ va $S_n^- \le S_n' < S$ (chegaralangan), demak, ular ham limitga ega:

$$\lim_{n\to\infty} S_n^+ = S^+, \quad \lim_{n\to\infty} S_n^- = S^-$$

$S_n = S_n^+ - S_n^-$ munosabatdan S_n ham limitga egaligi kelib chiqadi:

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} S_n^+ - \lim_{n\to\infty} S_n^- = S^+ - S^-$$

4-ta'rif. (18) va (19) qatorlar bir paytda yaqinlashuvchi bo'lsa, (18) qatorga absolyut yaqinlashuvchi qator deyiladi.

5-ta'rif. Agar (18) qator yaqinlashuvchi bo'lib (19) qator uzoqlashuvchi bo'lsa, u holda berilgan (18) qatorga shartli yaqinlashuvchi deyiladi.

1-Misol. Quyidagi qatorni ko'raylik:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots + (-1)^n \frac{1}{n} + \ldots$$

Leybnis alomatiga ko'ra bu qator yaqinlashuvchi, lekin qator hadlarining absolyut qiymatlaridan tuzilgan $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{n}+...$ qator esa uzoqlashuvchi. Demak, qator shartli yaqinlashuvchi.

2-Misol. Quyidagi qatorni ko'ramiz:

$$1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+...+\frac{(-1)^{n+1}}{n^2}+...$$

Yechish. Bu qator absolyut yaqinlashuvchidir, chunki u yaqinlashuvchidir va uning hadlari absolyut qiymatlaridan tuzilgan qator yaqinlashuvchidir (r=2>1).

3-Misol. O'zgaruvchan ishorali

$$\frac{\sin\alpha}{1^2}+\frac{\sin 2\alpha}{2^2}+...+\frac{\sin n\alpha}{n^2}+...$$

Qatorning yaqinlashishini tekshiring, buerda α -ixtiyoriy haqiqiy son.

Yechish. Berilgan qator bilan birga

$$\left|\frac{\sin\alpha}{1^2}\right|+\left|\frac{\sin 2\alpha}{2^2}\right|+...+\left|\frac{\sin n\alpha}{n^2}\right|+...$$

qatorni qaraymiz. Bu yaqinlashuvchi

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+...+\frac{1}{n^2}+...$$

qator bilan taqqoslaymiz.

Ravshanki, $\left|\frac{\sin n\alpha}{n^2}\right|\le\frac{1}{n^2}$ n=1,2,...

Shu sababli taqqoslash alomatiga ko'ra absolyut hadli qatorlar yaqinlashuvchi. U holda yuqorida isbotlangan teoremaga ko'ra berilgan qator yaqinlashuvchi.

Absolyut va shartli yaqinlashuvchi qatorlarning quyidagi xossalarini qayd qilamiz:

a) agar qator absolyut yaqinlashuvchi bo'lsa, u holda bu qator hadlarining o'rni har qancha almashtirilganda ham u absolyut yaqinlashuvchi bo'lib qolaveradi, bunda qatorning yig'indisi uning hadlari tartibiga bog'liq bo'lmaydi (bu xossa shartli yaqinlashuvchi qatorlar uchun o'rinli bo'lmasligi mumkin);

b) agar qator shartli yaqinlashuvchi bo'lsa, u holda bu qator hadlarining o'rinlarini shunday almashtirish mumkinki, natijada uning yig'indisi o'zgaradi va almashtirishdan keyin hosil bo'lgan qator uzoqlashuvchi qator bo'lib qolishi ham mumkin.

Misol uchun shartli yaqinlashuvchi

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\frac{1}{8}+...$$

qatorni olamiz. Uning yig'indisini S bilan belgilaymiz. Qator hadlarini har bir musbat haddan keyin ikkita manfiy had turadigan qilib almashtiramiz:

$$1-\frac{1}{2}-\frac{1}{4}+\frac{1}{3}-\frac{1}{6}-\frac{1}{8}+\ldots$$

Har bir musbat hadni undan keyin keladigan manfiy had bilan qo'shamiz:

$$\frac{1}{2}-\frac{1}{4}+\frac{1}{6}-\frac{1}{8}+\ldots$$

Natijada hadlari berilgan qator hadlarini $\frac{1}{2}$ ga ko'paytirishdan hosil bo'lgan qatorga ega bo'lamiz. U holda bu qator yaqinlashuvchi va uning yig'indisi $\frac{1}{2}S$ ga teng. Shunday qilib, qator hadlarining joylashish tartibini o'zgartirish bilangina uning yig'indisini ikki marta kamaytirdik.

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