Darajali qatorlar. Darajali qatorlarning yaqinlashish radiusi va sohasi. Teylor formulasi va qatori

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Annotatsiya: Ushbu maqolada Oliy matematikaning qiziqarli mavzularidan biri bo'lgan Darajali qatorlarning yaqinlashish radiusi va sohasi. Koshi-Adamar formulasi, darajali qatorlarning funksional xossalari haqida ma'lumotlar keltirildi hamda quyidagi muammolar xal etildi. Darajali qator tushunchasi. Abel teoremasi. Darajali qatorning yaqinlashish radiusi va yaqinlashish intervali. Bu hollarda qo'yilgan masalalarni yechishda quyida biz o'rganadigan qatorlar nazariyasi katta ahamiyatga ega.

Kalit so'zlar: Darajali qatorlar, Abelh teoremasi, Darajali qatorlarning yaqinlashish radiusi va intervali, Teylor formulasi va qatori, Teylor qatori, Ayrim funksiyalarni Makloren qatoriga yoyish, Binomial qator, Darajali qatorlarning taqribiy hisoblashlarga tatbiqi.

Graded rows. Radius and Area of Convergence of Level Lines. Taylor formula and series

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Abstract: In this article, one of the interesting topics of Higher Mathematics, the radius of convergence and the domain of Power Series. Information about Cauchy-Adamar formula, functional properties of graded series was given and the following problems were solved. The concept of a graded series. Abel's theorem. Convergence radius and convergence interval of a graded series. The theory of series, which we will study below, is of great importance in solving the problems posed in these cases.

Keywords: Power series, Abelh's theorem, Radius and interval of convergence of power series, Taylor's formula and series, Taylor series, Expansion of certain functions into Maclauren series, Binomial series, Application of power series to approximate calculations.

1-Ta'rif. Hadlari x o'zgaruvchining funksiyalardan iborat bo'lgan

$$u_1(x)+u_2(x)+\ldots+u_n(x)+\ldots \quad (1)$$

ko'rinishdagi qatorga funksional qator deyiladi.

Agar o'zgaruvchi x ning aniq bir qiymatini olsak ya'ni $x=x_0$ deb uni (1) ga qo'ysak $u_1(x_0)+u_2(x_0)+\ldots+u_n(x_0)+\ldots$ sonli qator hosil bo'ladi.

Demak o'zgaruvchi x ga aniq konkret har xil son qiymatlar berish bilan har xil yaqinlashuvchi yoki uzoqlashuvchi bo'lgan sonli qatorlar hosil qilish mumkin ekan.

2-Ta'rif. Agar (1) qator x ning $x_0,\ x_1,\ x_2,\ldots,x_n$ aniq son qiymatlarida yaqinlashuvchi bo'lsa u holda x ning bu $x_0,\ x_1,\ x_2,\ldots,x_n$ son qiymatlar to'plamiga (1) ning yaqinlashish sohasi deyiladi.

Misol. $1+x^2+x^3+\ldots+x^n+\ldots$ funksional qatorning hadlari mahraji $q=x$ ga teng bo'lgan geometrik progressiya tashkil qiladi.

Demak, uning yaqinlashishi uchun $|x|<1$ bo'lishi kerak va $(-1{,}1)$ intervalda qatorning yig'indisi $\frac{1}{1-x}$ ga teng. Shunday qilib, $(-1{,}1)$ intervalda berilgan qator

$$S(x)=\frac{1}{1-x}$$

funksiyani aniqlaydi, bu esa qatorning yig'indisidir, ya'ni

$$\frac{1}{1-x}=1+x^2+x^3+\ldots+x^n+\ldots$$

(1) Qatorning dastlabki n ta hadi yig'indisini $S_n(x)$ bilan belgilaylik:

$$S_n(x)=u_1(x)+u_2(x)+\ldots+u_n(x) \quad (2)$$

Agar $\lim\limits_{n\to\infty} S_n(x)=S(x)$ chekli limit mavjud bo'lsa (1) funksional qatorga yaqinlashuvchi qator deyilib $S(x)$ ga esa uning yig'indisi deyiladi.

Agar $\lim\limits_{n\to\infty} S_n(x)$ mavjud bo'lmasa (1) funksional qatorga uzoqlashuvchi deyiladi.

Agar bu qator x ning biror qiymatida yaqinlashsa, u holda

$$S(x)=S_n(x)+r_n(x)$$

bo'ladi, bu yerda

$S(x)$- qatorning yig'indisi $r_n(x)=u_{n+1}(x)+u_{n+2}(x)+\ldots$ - qatorning qoldig'i deyiladi.

x ning barcha qiymatlari uchun qatorning yaqinlashish sohasida

$$\lim_{n\to\infty} S_n(x)=S(x)$$

munosabat o'rinli, shu sababli $\lim\limits_{n\to\infty}[S(x)-S_n(x)]=0$ yoki $\lim\limits_{n\to\infty} r_n(x)=0$, ya'ni yaqinlashuvchi qatorning qoldig'i $n\to\infty$ da nolga intiladi.

1-Misol. Ushbu

$$\frac{\sin^2 x}{1^3}+\frac{\sin^2 2x}{2^3}+\ldots+\frac{\sin^2 nx}{n^3}+\ldots$$

funksional qator x ning barcha haqiqiy qiymatlari uchun tekis yaqinlashadi, chunki barcha x va n -larda

$$\left|\frac{\sin^2 nx}{n^3}\right|\leq\frac{1}{n^3}$$

$\frac{1}{1^3}+\frac{1}{2^3}+\ldots+\frac{1}{n^3}+\ldots$ qator esa yaqinlashuvchidir.

2-Misol. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n+x}$ qatorni tekshiring.

Veyershtrass alomati bu qator uchun bajarilmaydi, chunki berilgan qator shartli yaqinlashuvchi va $x\geq 0$ lar uchun $\sum_{n=1}^{\infty}\frac{1}{n+x}$ qator uzoqlashuvchi. Berilgan qatorni tekis yaqinlashuvchiligini ko'rsatish uchun Leybnis teoremasidan foydalanamiz. Berilgan qator o'zgaruvchi ishorali va $x\geq 0$ da absolyut qiymatlari bo'yicha monoton kamayuvchi va n -hadi $n\to\infty$ da nolga intiladi. SHu sababli, qator $[0,\infty)$ yarim o'qda yaqinlashuvchi va qator qoldig'i uchun $|r_n(x)|<\frac{1}{n+1+x}$ $x\geq 0$ da $|r_n(x)|\leq\frac{1}{n+1}$ ga ega bo'lamiz va $\lim_{n\to\infty}\frac{1}{n+1}=0$ bo'lgani uchun, qator tekis yaqinlashuvchi.

Tekis yaqinlashuvchi funksional qatorlar uchun funksiyalar chekli yig'indisi xossalarini tatbiq qilish mumkin.

1-teorema. Agar $u_1(x)+u_2(x)+\ldots+u_n(x)+\ldots$ funksional qatorning har bir hadi $[a,b]$ kesmada uzluksiz bo'lib, bu funksional qator $[a,b]$ kesmada tekis yaqinlashuvchi bo'lsa, u holda qatorning yig'indisi $S(x)$ ham shu kesmada uzluksiz bo'ladi.

3-Misol. $f(x)=\sum_{n=1}^{\infty}\left(x^2+\frac{1}{n}\right)^n$ funksiyani aniqlanish sohasini toping va uzluksizligini tekshiring.

Yechish. Berilgan funksional qatorni Koshi alomatiga ko'ra aniqlanish sohasini topamiz.

$$\lim_{n\to\infty}\sqrt[n]{\left(x^2+\frac{1}{n}\right)^n}=\lim_{n\to\infty}\left(x^2+\frac{1}{n}\right)=x^2$$

Shu sababli $x^2<1$ da qator yaqinlashuvchi va $x^2>1$ da uzoqlashuvchi, ya'ni qator (-1,1) oraliqda qator yaqinlashuvchi. $x=\pm 1$ nuqtalarda uzoqlashuvchi, chunki

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e\neq 0$$

qator yaqinlashishining zaruriy sharti bajarilmaydi.

Funksiyani uzluksizligini tekshiramiz. Buning uchun qatorni $0<a<1$ bo'lgan ixtiyoriy $[-a,a]$ kesmada tekis yaqinlashuvchi ekanligini ko'rsatamiz.

$0<a<b<1$ son olamiz va shunday N topiladiki, $n\geq N$ da $a+\frac{1}{\sqrt{n}}\leq b$. U holda $|x|\leq a$ lar uchun

$$\left(x^2+\frac{1}{n}\right)^n\leq\left(|x|+\frac{1}{\sqrt{n}}\right)^{2n}\leq\left(a+\frac{1}{\sqrt{n}}\right)^{2n}\leq b^{2n}$$

tengsizlik bajariladi.

Ravshanki, $b^2+b^4+b^6+\ldots+b^{2m}+\cdots$qator $[-a,a]$ da yaqinlashuvchi (chunki bu qator mahraji $b^2<1$ bo'lgan geometrik progressiya), shu sababli berilgan qator tekis yaqinlashuvchi. Demak, $f(x)$ funksiya $[-a,a]$ kesmada uzluksiz. a ($0<a<1$) ning ixtiyoriyligidan $f(x)$ funksiya (-1,1) oraliqda uzluksiz.

2-teorema. (Qatorlarni hadlab integrallash)

Agar $u_1(x)+u_2(x)+\ldots+u_n(x)+\ldots$funksional qatorning har bir hadi $[a,b]$ kesmada uzluksiz bo'lib, bu funksional qator $[a,b]$ kesmada tekis yaqinlashuvchi bo'lsa, u holda

$$\int_a^b S(x)dx=\int_a^b u_1(x)dx+\int_a^b u_2(x)dx+\ldots+\int_a^b u_n(x)dx+\ldots+$$

tenglik o'rinli bo'ladi.

$$S(x)=S_n(x)+r_n(x)=\underbrace{u_1(x)+u_2(x)+\ldots+u_n(x)}_{s_n(x)}+r_n(x)=S(x)-S_n(x)$$

Isbot.

(1) qator tekis yaqinlashuvchi qator bo'lgani uchun Veyershtrass teoremasidagi kabi

$$\lim_{n\to\infty}r_n(x)=0\Rightarrow\lim_{n\to\infty}\int_a^b r_n(x)dx=0$$

ekanligi ravshan.

$$\lim_{n\to\infty}\int_a^b r_n(x)dx=\lim_{n\to\infty}\int_a^b\left[\int_a^b S(x)dx-\int_a^b S_n(x)dx\right]\Rightarrow$$

$$\int_a^b S(x)dx=\int_a^b u_1(x)dx+\int_a^b u_2(x)dx+\ldots+\int_a^b u_n(x)dx+\ldots+$$

Teorema isbot bo'ldi.

4-Misol. $1-x^2+x^4-\ldots+(-1)^n x^{2n}+\ldots$ funksional qator $|x|<1$da tekis yaqinlashuvchi va uning yig'indisi $S(x)=\frac{1}{1+x^2}$ ga teng. Berilgan qatorni 0 dan x<1 gacha hadlab integrallaymiz va quyidagi qatorga ega bo'lamiz :

$$x-\frac{x^3}{3}+\frac{x^5}{5}-...+(-1)^n\frac{x^{2n+1}}{2n+1}+...$$

Bu qator qator $|x|<1$da tekis yaqinlashadi va uning yig'indisi quyidagiga teng:

$$\int_0^x S(x)dx=\int_0^x \frac{dx}{1+x^2}=arctg\Big|_0^x=arctgx$$

Shunday qilib $|x|<1$da tekis yaqinlashuvchi

$$arctgx=x-\frac{x^3}{3}+\frac{x^5}{5}-...+(-1)^n\frac{x^{2n+1}}{2n+1}+...$$

qatorga ega bo'ldik.

3-teorema. (Qatorlarni hadlab differensiallash)

Agar $[a,b]$ kesmada hosilalari uzluksiz bo'lgan funksiyalardan tuzilgan.

$$u_1(x)+u_2(x)+...+u_n(x)+...$$

funksional qator shu kesmada yaqinlashuvchi va yig'indisi $S(x)$ bo'lsa, u holda uning hadlarining hosilalaridan tuzilgan.

$$u_1'(x)+u_2'(x)+...+u_n'(x)+...$$

qator ham tekis yaqinlashuvchi bo'lib, yig'indisi $S'(x)$ bo'ladi.

5-Misol. 4- misolni qaraymiz: $arctgx=x-\frac{x^3}{3}+\frac{x^5}{5}-...+(-1)^n\frac{x^{2n+1}}{2n+1}+...$

Bundan x $arctgx=x^2-\frac{x^4}{3}+\frac{x^6}{5}-...+(-1)^n\frac{x^{2n+2}}{2n+1}+...$ ekani kelib chiqadi. Bunda o'ng tomonda biror qator turibdi. SHu qatorni hadlab differensiallab quyidagini topamiz:

$$2x-\frac{4x^3}{3}+\frac{6x^5}{5}-...+(-1)^n\frac{(2n+2)x^{2n+1}}{2n+1}+...$$

Dalamber alomatiga ko'ra

$$\lim_{n\to\infty}\frac{u_{n+1}}{u_n}=\lim_{n\to\infty}\left|\frac{\frac{2n+2}{2n+1}x^{2n+1}}{\frac{2n}{2n-1}x^{2n-1}}\right|=\lim_{n\to\infty}\frac{2(n+1)(2n-1)}{(2n+1)^2}x^2=x^2$$

Shunday qilib, qator absolyut yaqinlashuvchi va barcha $|x|<1$lar uchun tekis yaqinlashuvchi bo'ladi.

Demak, berilgan qatorning hosilalaridan tuzilgan qator berilgan qator yig'indisidan olingan hosilaga yaqinlashadi:

$$arctgx+\frac{x}{1+x^2}=2x-\frac{4x^3}{3}+\frac{6x^5}{5}-...+(-1)^n\frac{(2n+2)x^{2n+1}}{2n+1}+...$$

$|x|<1$ da tekis yaqinlashuvchidir.

Teylor qatori

Biz birinchi kurs materiallaridan bilamizki, agar $f(x)$ funksiya $x=a$ nuqtani o'z ichiga olgan biror intervalda $n+1$ -tartibli hamma hosilalarga ega bo'lsa, bu funksiya uchun $x=a$ nuqta atrofida quyidagi Teylor formulasi o'rinli bo'lar edi:

$$f(x)=f(a)+\frac{x-a}{1!}f'(a)+\ldots+\frac{(x-a)^n}{n!}f^{(n)}(a)\ldots \quad (1)$$

Qoldiq had $R_n(x)$ esa (Lagranj ko'rinishidagi)

$$R_n(x)=\frac{(x-a)^{n+1}}{(n+1)!}f^{(n+1)}[a+0(x-a)] \qquad 0<0<1$$

formula bilan hisoblanar edi.

Faraz qilaylik $n\to\infty$ da $R_n(x)\to 0$ bo'lsin, ya'ni

$$\lim_{n\to\infty}R_n(x)=\lim_{n\to\infty}\left[\frac{(x-a)^{n+1}}{(n+1)!}f^{(n+1)}[a+0(x-a)]\right]=0$$

bo'lsin.

$f(x)$ funksiya $x=a$ nuqta atrofida hamma hosilalari mavjud bo'lgani uchun n ni etarli darajada katta qilib olishimiz mumkin, ya'ni $n\to\infty$ desak

$$f(x)=f(a)+\frac{x-a}{1!}f'(a)+\ldots+\frac{(x-a)^n}{n!}f^{(n)}(a)\ldots \quad (2)$$

hosil bo'ladi. (2) ga Teylor qatori deyiladi.

(2) formula faqat $n\to\infty$ da $R_n(x)=0$ bo'lgandagina o'rinli bo'lib, bu holda (2) qatorga yaqinlashuvchi qator deyilib $f(x)$ ga esa uning yig'indisi deyiladi.

Haqiqatdan

$$f(x)=f(a)+\frac{x-a}{1!}f'(a)+\ldots+\frac{(x-a)^n}{n!}f^{(n)}(a)\ldots+R_n(x)=P_n(x)+R_n(x)$$

$$\lim_{n\to\infty}f(x)=\lim_{n\to\infty}P_n(x)+\lim_{n\to\infty}R_n(x) \quad \Rightarrow f(x)=\lim_{n\to\infty}P_n(x)$$

Agar Teylor qatorida $a=0$ desak

$$f(x)=f(0)+\frac{x}{1!}f'(0)+\ldots+\frac{x^n}{n!}f^{(n)}(0)\ldots \quad (3)$$

Makloren qatori kelib chiqadi.

Endi shunday savolning tug'ilishi tabiiy: Qanday funksiyalar Teylor qatoriga yoyiladi?

Bu savola quyidagi teorema javob beradi:

4-Teorema. $f(x)$ funksiya $(-r,r)$ intervalda aniqlangan bo'lib, unda noldan farqli istalgan tartibli hosilalari mavjud bo'lsin.

Agar shunday bir M soni mavjud bo'lsaki, $(-r,r)$ intervalning barcha nuqtalarida $\left|f^{(n)}(x)\right|<M,\ (n=0,1,2,..)$

Tengsizlik o'rinli bo'lsa, u holda intervalda

$$f(x)=f'(0)+\frac{f'(0)}{1!}x+\frac{f^{(n)}(0)}{2!}+...+\frac{f^{(n)}(0)}{n!}x^n+... \quad (3)$$

tenglik o'rinli bo'ladi.

Shunday qilib bu teoremaga asosan $f(x)$ funksiyaning istalgan tartibli hosilalari mavjud bo'lib ularning hammasi yuqoridan chegaralangan bo'lsa $f(x)$ funksiya uchun (3) yoyilma o'rinli bo'lar ekan.

Ayrim funksiyalarni Makloren qatoriga yoyish.

1. $f(x)=\sin x$ funksiyani Makloren qatoriga yoyaylik.

$f(x)=\sin x$ funksiya yuqoridagi teorema shartlarini har qanday r uchun ya'ni ixtiyoriy $(-r,r)$ intervalda qanoatlantiradi.

Haqiqatan, $\sin x$ funksiyaning istalgan tartibli hosilasi yoki $\pm\sin x$ ga yoki $\pm\cos x$ ga teng; ikkinchidan $\left|(\sin x)^n\right|\le 1,\ \left|(\cos x)^n\right|\le 1\ \ (n=0,1,2,..)$

$f(x)=\sin x$, $f'(x)=(\sin x)'=\cos x,\ \ f''(x)=(\sin x)''=-\sin x,$

$f'''(x)=(\sin x)```=-\cos x,$

$f^{IV}(x)=(\sin x)^{IV}=\sin x,...$

Bundan ko'rinadiki $\{(\sin x)^{(n)}\}$ ketma-ketlik davriy bo'lib davri 4 ga teng ekan.

Agar $x=0$ desak

$\sin 0=0,\ \ \sin'(0)=1,\ \ \sin''(0)=0,\ \ \sin'''(0)=-1,...,\sin^{(2n+1)}(0)=(-1)^n\ \ (n=0,1,2,..)$

Endi buni (3) ga qo'ysak

$$\sin x=x-\frac{x^3}{3!}+...+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+... \quad (4)$$

2. Xuddi shuningdek $f(x)=\cos x$ funksiyani Makloren qatoriga yoyishimiz mumkin.

$f(x)=\cos x$ funksiyaning istalgan tartibli hosilasi yoki $\pm\sin x$ ga yoki $\pm\cos x$ ga teng; ikkinchidan $\left|(\sin x)^n\right|\le 1,\ \left|(\cos x)^n\right|\le 1\ \ (n=0,1,2,..)$

$f(x)=\cos x$, $f'(x)=(\cos x)'=-\sin x,\ \ f''(x)=(\cos x)''=-\cos x,$

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$f^{IV}(x)=(\cos x)^{IV}=\cos x.$

Bundan ko'rinadiki $\{(\cos x)^{(n)}\}$ ketma-ketlik davriy bo'lib davri 4 ga teng ekan.

Agar $x=0$ desak

$\cos 0=1, \cos'(0)=0, \cos''(0)=-1, \cos'''(0)=0,...,\cos^{(2n)}(0)=(-1)^n, \cos^{(2n+1)}(0)=0\ (n=0,1,2,...)$

Endi buni (3) ga qo'ysak

$$\cos x = 1 - \frac{x^2}{2!} + ... + (-1)^n \frac{x^{2n}}{(2n)!} \quad (5)$$

3) $f(x) = e^x$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^n}{n!} + ... \quad (6)$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + ... + (-1)^n \frac{x^n}{n!} + ... \quad (7)$$

Bu (4)-(7) formulalarni $\sin x,\ \cos x,\ e^x$ funksiyalarni Teylor formulasiga yoyilmalaridan bevosita $n \to \infty$ da $R_n(x) \to 0$ deb to'g'ridan-to'g'ri yozib qo'yish ham mumkin:

1) $f(x) = \sin x$ bo'lsin.

$$\sin x = x - \frac{x^3}{3!} + ... + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + R_n(x),\ (n = 0,1,2,...)$$

$\lim_{n\to\infty} R_n(x) = 0$ desak (4) kelib chiqadi.

2) $f(x) = \cos x$ bo'lsin.

$$\cos x = 1 - \frac{x^2}{2!} + ... + (-1)^n \frac{x^{2n}}{(2n)!} + R_n(x),\ (n = 0,1,2,...)$$

Bu erda ham $\lim_{n\to\infty} R_n(x) = 0$ desak (5) kelib chiqadi.

Misol. $\sin x$ ning $x = 10^0$ dagi qiymatini hisoblang.

$x = 10^0 = \frac{\pi}{18} \approx 0,174533$ radianda

$$\sin x = \sin\frac{\pi}{18} = \frac{\pi}{18} - \frac{1}{3!}\left(\frac{\pi}{18}\right)^3 + \frac{1}{5!}\left(\frac{\pi}{18}\right)^3 - ... \approx \frac{\pi}{18} - \frac{1}{3!}\left(\frac{\pi}{18}\right)^3 \approx 0,173647$$

Binomial qator

$$f(x) = (1 + x)^m \quad (8)$$

funksiyani Makloren qatoriga yoyaylik. m noldan va barcha natural sonlardan farqli ixtiyoriy o'zgarmas haqiqiy son.

Agar m natural son bo'lsa, bizga ma'lum bo'lgan Nqyuton formulasi ya'ni chekli yoyilma hosil bo'ladi.

Bu erda (8) funksiyaning qoldiq hadini baholash ancha qiyinchilik tug'diradi.

Shuning uchun biz quyidagicha ish ko'ramiz.

(8) funksiya

$$(1 + x) f'(x) = m f(x) \quad (9)$$

differnsial tenglamani va

$$f(0)=1 \quad (10)$$

boshlang'ich shartni qanoatlantiradi. Endi shunday bir darajali qator olaylikki u (9) va (10) ni qanotlantirib, yaqinlashuvchi bo'lsin va yig'indisi $S(x)$ bo'lsin, ya'ni

$$S(x)=1+a_1x+a_2x^2+...+a_nx^n+... \quad (11)$$

(10) ni (8) ga qo'ysak

$$(1+x)(a_1+2a_2x+...+na_nx^{n-1}+...)=m(1+a_1x+a_2x^2+...+a_nx^n+...)$$

$$a_1+(a_1+2a_2)x+(2a_2+3a_3)x^2+...+(na_n+(n+1)a_{n+1})x^n+...=$$

$$=m(1+a_1x+a_2x^2+...+a_nx^n+...)$$

Endi bir xil darajali x larning oldidagi koeffisientlarni tenglashtirsak:

$$\left.\begin{array}{l} a_1=m \\ a_1+2a_2=ma_1 \\ 2a_2+3a_3=ma_2 \\ \dots\dots\dots\dots \\ na_n+(n+1)a_{n+1}=ma_n \end{array}\right\} \Rightarrow \left\{\begin{array}{l} a_1=m, a_2=\dfrac{m(m-1)}{2!}, a_3=\dfrac{m(m-1)(m-2)}{2\cdot 3} \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ a_n=\dfrac{m(m-1)(m-2)...(m-n+1)}{1\cdot 2\cdot 3...\cdot n} \\ \dots\dots\dots\dots\dots\dots\dots\dots \end{array}\right.$$

Hosil qilingan (12) koeffisientlar binomial koeffisentlar deyiladi. (12) ni (11) ga qo'ysak.

$$S(x)=1+mx+\frac{m(m-1)}{2!}x^2+...+\frac{m(m-1)(m-2)...(m-n+1)}{n!}x^n+... \quad (13)$$

Biz bilamizki biror differensial tenglamaning echimi mavjud bo'lsa va u berilgan boshlang'ich shartni qanoatlantirsa, bunday echim yagona bo'ladi. Shuning uchun (10) va (11) ni ham $(1+x)^m$ va $S(x)$ lar qanoatlantirgani uchun ular aynan teng bo'lishi kerak va

$$S(x)=(1+x)^m \Rightarrow (1+x)^m=1+mx+\frac{m(m-1)}{2!}x^2+...+\frac{m(m-1)(m-2)...(m-n+1)}{n!}x^n+... \quad (13)$$

munosabat o'rinli bo'ladi. (13) ga binomial qator deyiladi.

Endi (13) ning yaqinlashish radiusini topaylik.

$$u_n=\frac{m(m-1)...(m-n+2)}{(n-1)!}x^{n-1};\ u_{n+1}=\frac{m(m-1)...(m-b+1)}{n!}x^n$$

$$\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right|=\lim_{n\to\infty}\left|\frac{\frac{m(m-1)...(m-n+1)}{n!}x^n}{\frac{m(m-1)...(m-n+2)}{(n-1)!}x^{n-1}}\right|=\lim_{n\to\infty}\left|\frac{m-n+1}{n}\right||x|<1\Rightarrow |x|<1\Rightarrow(-1,1)$$

Shunday qilib (13) faqat (-1,1) da o'rinli.

(13) dagi m ga har xil manfiy va kasr qiymatlar berib har xil funksiyalarning darajali qatorga yoyilmalarini hosil qilamiz.

$m=-1$ da $\frac{1}{1+x}=1-x+x^2-x^3+...+(-1)^n x^n+...$ (14)

$m=\frac{1}{2}$ da $\sqrt{1+x}=1+\frac{1}{2}x-\frac{1}{2\cdot 4}x^2+\frac{1\cdot 3}{2\cdot 4\cdot 6}x^3-...$

Darajali qatorlarning taqribiy hisoblashlarga tatbiqi.

1. $f(x)=ln(1+x)$ funksiyani darajali qatorga yoyishni ko'raylik.

$$\frac{1}{1+x}=1-x+x^2-x^3+...+(-1)^n x^n+... \quad (15)$$

(-1,1) da o'rinli bo'lgani uchun, ya'ni $|x|<1$ bo'lgani uchun $[0,x]$ kesmada hadma-had integrallasak

$$\int_0^x \frac{dx}{1+x}=\int_0^x\left(1-x+x^2-x^3+...+(-1)^n x^n+...\right)dx \Rightarrow$$

$$ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-...+(-1)^{n+1}\frac{x}{n+1}+... \qquad x\in(-1,1) \quad (16)$$

x ni –x ga almashtirsak

$$ln(1-x)=-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}-... \qquad x\in(-1,1) \quad (17)$$

Agar (16) da $x=1$ desak

$ln\,2=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...$

(16) dan (17) ni ayirsak

$$ln\frac{(1+x)}{(1-x)}=2\left(x+\frac{x^3}{3}+\frac{x^5}{5}+\frac{x^7}{7}+...\right) \quad (18)$$

Logarifmlarni hisoblash uchun qulay bo'lgan formula kelib chiqadi.

Masalan, $ln\,3-?$

$\frac{(1+x)}{(1-x)}=3$ desak $x=\frac{1}{2}<1$ bo'lgani uchun (18) dan foydalansak

$ln\,3=2\left(\frac{1}{2}+\frac{1}{3}\left(\frac{1}{2}\right)^3+\frac{1}{5}\left(\frac{1}{2}\right)^5+\frac{1}{7}\left(\frac{1}{2}\right)^7+...\right)$

ni hosil qilamiz va bu formuladagi xatolik har vaqt $\Delta_n=\frac{2x^{2n+1}}{(2n+1)(1-x^2)}$ dan kichik bo'ladi.

2. $f(x)=arc\,tg\,x$ funksiyani Makloren qatoriga yoyilsin.

(15) formula (-1,1) da o'rinli bo'lgani uchun ch ni x^2 bilan almashtirib, so'ngra $[0,x]$ da (-1,1) da integrallaymiz:

$\frac{1}{1+x^2}=1-x^2+x^4-x^6+x^8-x^{10}+... \qquad (-1,1)$ da

$$\int_0^x \frac{dx}{1+x^2} = \int_0^x \left(1 - x^2 + x^4 - x^6 + \ldots\right)dx \Rightarrow arctgx = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \ldots |x| < 1$$

Bu qator (-1,1) da yaqinlashuvchi bo'lib yig'indisi $arc\,tg\,x$ bo'ladi. Xatto $x = \pm 1$ da ham o'rinli ekanligini ko'rsatish mumkin.

3. Ildizlarni taqribiy hisoblash.

Masalan, $\sqrt[4]{650}$ ni $0{,}001$ aniqlikda hisoblash kerak.

$5^4 = 625 < 650;\ 6^4 = 1296 > 650.$ Demak 650 ning butun qismi 5 ga teng.

$$\sqrt[4]{650} = \sqrt[4]{625 + 25} = \sqrt[4]{5^4\left(1 + \frac{25}{625}\right)} = 5\left(1 + \frac{1}{25}\right)^{\frac{1}{4}}$$

Endi

$$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!}x^2 + \ldots + \frac{m(m-1)(m-2)\ldots(m-n+1)}{n!}x^n + \ldots$$

binomial qatordan foydalansak

$$\sqrt[4]{650} = 5\left(1 + \frac{1}{25}\right)^{\frac{1}{4}} = 5\left[1 + \frac{1}{4}\cdot\frac{1}{25} + \frac{\frac{1}{4}\left(\frac{1}{4}-1\right)}{1\cdot 2}\cdot\frac{1}{25^2} + \frac{\frac{1}{4}\left(\frac{1}{4}-1\right)\left(\frac{1}{4}-2\right)}{1\cdot 2\cdot 3}\cdot\frac{1}{25^3} + \ldots\right] \approx$$

$$\approx 5\left[1 + \frac{1}{4\cdot 25} + \frac{\frac{1}{4}\left(\frac{1}{4}-1\right)}{1\cdot 2}\cdot\frac{1}{25^2}\right].$$

Bundan xatolik absolyut jihatdan

$$\frac{\frac{1}{4}\left(\frac{1}{4}-1\right)\left(\frac{1}{4}-2\right)}{1\cdot 2\cdot 3}\cdot\frac{1}{25^3} = \frac{1\cdot 3\cdot 7}{1\cdot 2\cdot 3}\cdot\frac{1}{100^3} = 0{,}0000035$$ dan oshmaydi.

4. Qatorlar yordamida aniq integrallarni taqribiy hisoblash.

Agar ixtiyoriy $f(x)$ funksiya (a,b) da uzluksiz bo'lsa, bu funksiya shu intervalda boshlang'ich funksiyaga ega bo'ladi ya'ni $f(x) = F'(x)$ bo'ladi. Lekin ba'zi hollarda boshlang'ich funksiyani elementar funksiyalar orqali ifodalash mumkin bo'lavermaydi.

Masalan,

$$\int_0^x e^{-x^2}dx, \qquad \int_0^x \frac{\sin x}{x}dx, \qquad \int_0^x \frac{\cos x}{x}dx, \qquad \int_0^x \frac{dx}{\ln x}dx$$

kabi integrallar bilan ifodalangan boshlang'ich funksiyalarni elementar funksiyalar orqali ifodalab bo'lmaydi.

a) $\int_0^x e^{-x^2}dx,$ integralni ko'raylik

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots + (-1)^n\frac{x^n}{n!} + \ldots, \qquad (-\infty, \infty)$$

x ni x^2 bilan almashtirsak

$$e^{-x}=1-x^2+\frac{x^4}{2!}-\frac{x^6}{3!}+...+(-1)^n\frac{x^{2n}}{n!}+...,\qquad(-\infty,\infty)$$

$$\int_0^x e^{-x^2}dx=\int_0^x\left(1-x^2+\frac{x^4}{2!}-\frac{x^6}{3!}+...+(-1)^n\frac{x^{2n}}{n!}+...\right)dx=x-\frac{x^3}{3}+\frac{x^5}{5\cdot 2!}-\frac{x^7}{7\cdot 3!}+...$$

Agar $[0,1]$ olsak

$$\int_0^x e^{-x^2}dx=1-\frac{1}{3}+\frac{1}{5\cdot 2!}-\frac{1}{7\cdot 3!}+...(-1)^n\frac{1}{n!}\cdot\frac{1}{2n+1}+...$$

b) $\int_0^x\frac{\sin x}{x}dx$ ni ko’raylik.

$$\frac{\sin x}{x}=\frac{x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+...}{x}=1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+...$$

$$\int_0^x\frac{\sin x}{x}dx=\int_0^x\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+...\right)dx=x-\frac{x^3}{3\cdot 3!}+\frac{x^5}{5\cdot 5!}-\frac{x^7}{7\cdot 7!}+...$$

s) $\int_0^x\frac{\cos x}{x}dx$ ham shunday integrallanadi.

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