Teylor qatori. Elementar funksiyalarni darajali qatorlarga yoyish

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Annotatsiya: Ushbu maqolada Oliy matematikaning qiziqarli mavzularidan biri bo'lgan Teylor qatori hamda Elementar funksiyalarni darajali qatorlarga yoyish haqida ma'lumotlar keltirildi va mavjud muammolar xal etildi. Funksiyaning Teylor qatori, Funksiyani Teylor qatoriga yoyish, Elementar funksiyalarni Teylor qatoriga yoyish. Bu hollarda qo'yilgan masalalarni yechishda quyida biz o'rganadigan qatorlar nazariyasi katta ahamiyatga ega.

Kalit so'zlar: Funksiyani Teylor qatoriga yoyish, Elementar funksiyalarni Teylor qatoriga yoyish.

Taylor line. Expansion of elementary functions into graded series

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Abstract: In this article, one of the interesting topics of Higher Mathematics, Taylor's series and expansion of elementary functions into graded series, is presented, and existing problems are solved. Taylor series of a function, Taylor series expansion of a function, Taylor series expansion of elementary functions. The theory of series, which we will study below, is of great importance in solving the problems posed in these cases.

Keywords: Expanding a function into a Taylor series, Expanding elementary functions into a Taylor series.

1^0. Funksiyaning Teylor qatori. Aytaylik, $f(x)$ funksiya $x_0 \in R$ nuqtaning biror

$$U_\delta(x_0) = \{x \in R : x_0 - \delta < x < x_0 + \delta; \delta > 0\}$$

atrofida isalgan tartibdagi hosilaga ega bo'lsin. Bu hol $f(x)$ funksiyaning Teylor formulasini yozish imkonini beradi:

$$f(x)= f(x_0)+\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+...+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n+r_n(x),$$

bunda $r_n(x)$-qoldiq had.

Modomiki, $f(x)$ funksiya $U_\delta(x_0)$ da isalgan tartibdagi hosilaga ega ekan, unda

$$f(x_0)+\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+...+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n+... \quad (1)$$

darajali qatorni qarash mumkin bo'ladi.

(1) darajali qatorning koeffisientlari sonlar bo'lib, ular $f(x)$ funksiya va uning hosilalarining x_0 nuqtadagi qiymatlari orqali ifodalangan.

(1) darajali qator $f(x)$ funksiyaning Teylor qatori deyiladi.

Xususan, $x_0 = 0$ bo'lganda (1) darajali qator ushbu

$$f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+...+\frac{f^{(n)}(0)}{n!}x^n+...=\sum_{n=1}^{\infty}\frac{f^{(n)}(0)}{n!}x^n$$

ko'rinishga keladi.

Faraz qilaylik, $f(x)$ funksiya biror $(-r,r)$ da $(r>0)$ isalgan tartibdagi hosilaga ega bo'lib, uning $x_0 = 0$ nuqtadagi Teylor qatori

$$f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+...+\frac{f^{(n)}(0)}{n!}x^n+... \quad (2)$$

bo'lsin. Bu qatorning qoldiq hadini $r_n(x)$ deylik:

$$f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+...+\frac{f^{(n)}(0)}{n!}x^n+r_n(x).$$

1-teorema. (2) darajali qator $(-r,r)$ da $f(x)$ ga yaqinlashishi uchun ushbu

$$f(x)= f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+...\frac{f^{(n)}(0)^n}{n!}+r_n(x)$$

Teylor formulasida, $\forall x\in(-r,r)$ uchun

$$\lim_{n\to\infty} r_n(x)=0$$

bo'lishi zarur va etarli.

Zarurligi. Aytaylik, (2) darajali qator $(-r,r)$ da yaqinlashuvchi, yi\indisi $f(x)$ bo'lsin. Ta'rifga binoan

$$\lim_{n\to\infty} S_n(x)= f(x), \qquad (x\in(-r,r))$$

bo'ladi, bunda

$$S_n(x)= f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+...\frac{f^{(n)}(0)^n}{n!}.$$

Ravshanki, $\forall x\in(-r,r)$ da $\lim\limits_{n\to\infty}S_n(x)=f(x)$ bo'lishidan

$$\lim_{n\to\infty}[f(x)-S_n(x)]=\lim_{n\to\infty}r_n(x)=0$$

bo'lishi kelib chiqadi.

Etarliligi. Aytaylik, $\forall x\in(-r,r)$ da $\lim\limits_{n\to\infty}r_n(x)=0$ bo'lsin. U holda

$$\lim_{n\to\infty}[f(x)-S_n(x)]=\lim_{n\to\infty}r_n(x)=0$$

bo'lib, undan

$$\lim_{n\to\infty}S_n(x)=f(x)$$

bo'lishi kelib chiqadi. Demak,

$$f(x)= f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+...\frac{f^{(n)}(0)^n}{n!}+...$$

bo'ladi.

Odatda, bu munosabat o'rinli bo'lsa, $f(x)$ funksiya Teylor qatoriga yoyilgan deyiladi.

2[0]. Funksiyani Teylor qatoriga yoyish. Faraz qilaylik, $f(x)$ funksiya biror $(-r,r)$ da isalgan tartibdagi hosila-larga ega bo'lsin.

2-teorema. Agar $\exists M>0,\ \forall x\in(-r,\ r),\ \forall n\ge 0$ da

$$\left|f^{(n)}(x)\right|\le M$$

bo'lsa, $f(x)$ funksiya $(-r,r)$ da Teylor qatoriga yoyiladi:

$$f(x)=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+...\frac{f_{(0)}^{(n)}}{n!}x^n+... \qquad (3)$$

Ma'lumki, $f(x)$ funksiyaning Lagranj ko'rinishidagi qoldiq hadli Teylor formulasi quyidagicha bo'ladi:

$$f(x)= f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+...\frac{f_{(0)}^{(n)}}{n!}x^n+r_n(x),$$

bunda,

$$r_n(x)=\frac{f^{(n)}(\theta x)}{(n+1)!}x^{n+1}. \qquad (0<\theta<1).$$

Teoremaning shartidan foydalanib topamiz:

$$|r_n(x)| = \left| \frac{f^{(n)}(\theta x)}{(n+1)!} x^{n+1} \right| \le M \cdot \frac{r^{n+1}}{(n+1)!} . \qquad (x \in (-r, r)).$$

Ravshanki,

$$\lim_{n\to\infty} \frac{r^{n+1}}{(n+1)!} = 0 .$$

Demak, $\forall x \in (-r, r)$ da

$$\lim_{n\to\infty} r_n(x) = 0$$

bo'lib, undan qaralayotgan $f(x)$ funksiyaning Teylor qatoriga yoyilishi kelib chiqadi.

3⁰. Elementar funksiyalarni Teylor qatoriga yoyish.

a) Ko'rsatkichli va giperbolik funksiyalarni Teylor qatorlarini topamiz. Aytaylik,

$$f(x) = e^x$$

bo'lsin. Ravshanki, $f(0)=1,\ f^{(n)}(0)=1 \qquad (n \in N)$ bo'lib, $\forall x \in (-\alpha, \alpha)$ da $(\alpha > 0)$

$$0 < f(x) < e^{\alpha},\ \ 0 < f^{(n)}(x) < e^{\alpha}$$

bo'ladi. Binobarin, 2-teoremaga ko'ra $f(x) = e^x$ funksiya $(-\alpha, \alpha)$ da Teylor qatoriga yoyiladi va (3) formulada foydalanib topamiz:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + \ldots \quad (0! = 1). \ (4)$$

$\alpha > 0$ ixtiyoriy musbat son. Demak, (4) darajali qatorning yaqinlashish radiusi $r = +\infty$ bo'ladi.

(4) munosabatda x ni $-x$ ga almashtirib topamiz:

$$e^{-x} = \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \ldots + (-1)^n \cdot \frac{x^n}{n!} + \ldots$$

Ma'lumki giperbolik sinus hamda giperbolik kosinus funksiyalari quyidagicha

$$shx = \frac{e^x - e^{-x}}{2},\ chx = \frac{e^x + e^{-x}}{2}$$

ta'riflanar edi.

Yuqoridagi

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots + \frac{x^n}{n!} + \ldots ,$$

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \ldots + (-1)^n \frac{x^n}{n!} + \ldots$$

formulalardan foydalanib topamiz:

$$shx=\frac{x}{1!}+\frac{x^3}{3!}+...+\frac{x^{2n+1}}{(2n+1)!}+...=\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!},$$

$$chx=1+\frac{x^2}{2!}+\frac{x^4}{4!}+...+\frac{x^{2n}}{(2n)!}+...=\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}.$$

Bu $shx,\ chx$ funksiyalarining Teylor qatorlari bo'lib, ular ifodalangan darajali qatorlarning yaqinlashish radiuslari $r=+\infty$ bo'ladi.

b) Trigonometrik funksiyalarning Teylor qatorlarini topamiz. Aytaylik, $f(x)=\sin x$ bo'lsin. Ravshanki, $\forall x\in R,\ \forall n\in N$ da

$$|f(x)|\le 1,\quad |f^{(n)}(x)|\le 1$$

bo'lib, $f(0),\ f'(0)=1,\ f^{(2n)}(0)=0,\ f^{(2n+1)}(0)=(-1)^n\ \ (n\in N)$ bo'ladi. Demak, 2-teoremaga ko'ra $f(x)=\sin x$ funksiya Teylor qatoriga yoyiladi va (3) formulaga binoan

$$\sin x=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1}=x-\frac{1}{3!}x^3+\frac{1}{5!}x^5-... \qquad (5)$$

bo'ladi.

Aytaylik,

$$f(x)=\cos x$$

bo'lsin. Bu funksiya uchun $\forall x\in R,\ \forall n\in N$ da

$$|f(x)|\le 1,\quad |f^{(n)}(x)|\le 1$$

bo'lib,

$$f(0)=1,\ f'(0)=0,\ f^{(2n)}(0)=(-1)^n,\ f^{(2n+1)}(0)=0\quad (n\in N)$$

bo'ladi. Unda 2–teoremaga ko'ra $f(x)=\cos x$ funksiya Teylor qatoriga yoyiladi va (3) formulaga binoan

$$\cos x=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}=1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-... \qquad (6)$$

bo'ladi.

(5) va (6) darajali qatorlarning yaqinlashish radiusi $r=+\infty$ bo'ladi.

v) Logarifmik funksiyaning Teylor qatorini topamiz. Aytaylik,

$$f(x)=\ln(1+x)$$

bo'lsin. Ma'lumki,

$$f^{(n)}(x)=\frac{(-1)^{n-1}(n-1)!}{(1+x)^n}\qquad (n\in N)$$

bo'lib,

$$\frac{f^{(n)}(0)}{n!}=\frac{(-1)^{n-1}}{n}$$

bo'ladi. Bu funksiyaning Teylor formulasi

$$\ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+...+(-1)^{n-1}\frac{x^n}{n}+r_n(x) \quad (7)$$

ko'rinishga ega.

$f(x)=\ln(1+x)$ funksiyani Teylor qatoriga yoyishda 1-teoremadan foydalanmiz. Buning uchun (7) formulada $r_n(x)$ ning 0 ga intilishini ko'rsatish etarli bo'ladi.

Aytaylik, $x\in[0,1]$ bo'lsin. Bu holda Lagranj ko'rinishida yozilgan

$$r_n(x)=\frac{(-1)^n x^{n+1}}{(n+1)(1+\theta x)^{n+1}} \quad (0<\theta<1)$$

qoldiq had uchun

$$|r_n(x)|\le\frac{1}{n+1}$$

bo'ladi va

$$\lim_{n\to\infty} r_n(x)=0$$

tenglik bajariladi.

Aytaylik, $x\in[-\alpha,0]$ bo'lsin, bunda $0<\alpha<1$.

Bu holda Koshi ko'rinishida yozilgan

$$r_n(x)=\frac{(-1)^n(1-\theta_1)^n\cdot x^{n+1}}{(1+\theta_1 x)^{n+1}} \qquad (0<\theta_1<1)$$

qoldiq had uchun

$$|r_n(x)|\le\frac{\alpha^{n+1}}{1-\alpha}$$

bo'lib,

$$\lim_{n\to\infty} r_n(x)=0$$

bo'ladi.

Demak, $\forall x\in(-1,1]$

$$\lim_{n\to\infty} r_n(x)=0.$$

Unda 1-teoremaga ko'ra

$$\ln(1+x)=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}x^n = x-\frac{x^2}{2}+\frac{x^3}{3}-...+(-1)^{n-1}\frac{x^n}{n}+... \quad (8)$$

bo'ladi.

(8) darajali qatorning yaqinlashish radiusi $r=1$ ga teng.

Agar yuqoridagi $\ln(1+x)$ ning yoyilmasida x ni $-x$ ga almashtirilsa, unda

$$\ln(1-x)=-\sum_{n=1}^{\infty}\frac{x^n}{n}=-x-\frac{x^2}{2}-\frac{x^3}{3}-...-\frac{x^n}{n}-...$$

formula kelib chiqadi.

g) Darajali funksiyaning Teylor qatorini topamiz.

Aytaylik,

$$f(x)=(1+x)^{\alpha} \qquad (\alpha\in R)$$

bo'lsin. Ma'lumki,

$$f^{(n)}(x)=\alpha(\alpha-1)(\alpha-2)...(\alpha-n+1)(1+x)^{\alpha-n} \quad (n\in N)$$

bo'lib,

$$f^{(n)}(0)=\alpha(\alpha-1)(\alpha-2)...(\alpha-n+1)$$

bo'ladi. Bu funksiyaning Teylor formulasi ushbu

$$(1+x)^{\alpha}=1+\frac{\alpha}{1!}x+\frac{\alpha(\alpha-1)}{2!}x^2+...+\frac{\alpha(\alpha-1)...(\alpha-n+1)}{n!}x^n+r_n(x)$$

ko'rinishga ega.

Endi $n\to\infty$ da $r_n(x)\to 0$ bo'lishini ko'rsatamiz.

Ma'lumki, Teylor formulasidagi qoldiq hadning Koshi ko'rinishi quyidagicha

$$r_n(x)=\frac{(\alpha-1)(\alpha-2)...[(\alpha-1)-(n-1)]}{n!}x^n\alpha\cdot x(1+\theta x)^{\alpha-1}\left(\frac{1-\theta}{1+\theta x}\right)^n$$

$(0<\theta<1)$ bo'lar edi.

Aytaylik, $x\in(-1,1)$ bo'lsin. Bu holda:

1) $\lim\limits_{n\to\infty}\frac{1}{n!}(\alpha-1)(\alpha-2)...[(\alpha-1)-(n-1)]x^n=0$ bo'ladi,

chunki, limit ishorasi osidagi ifoda yaqinlashuvchi ushbu

$$1+\sum_{n=1}^{\infty}\frac{\alpha(\alpha-1)...(\alpha-n+1)}{n!}x^n$$

qatorning umumiy hadi;

2) $|\alpha\cdot x|(1-|x|)^{\alpha-1}<\alpha\cdot x(1+\theta x)^{\alpha-1}<|\alpha\cdot x|(1+|x|)^{\alpha-1}$;

3) $\left|\frac{1-\theta}{1+\theta x}\right|^n \le \left|\frac{1-\theta}{1+\theta x}\right| < 1$

bo'ladi. Bu munosabatlardan foydalanib, $\forall x \in (-1,1)$ da

$$\lim_{n\to\infty} r_n(x) = 0$$

bo'lishini topamiz. 1-teoremaga ko'ra

$$(1+x)^\alpha = 1 + \frac{\alpha}{1!}x + \frac{\alpha(\alpha-1)}{2!}x^2 + \ldots + \frac{\alpha(\alpha-1)\ldots(\alpha-n+1)}{n!}x^n + \ldots \quad (9)$$

bo'ladi.

Bu darajali qatorning yaqinlashish radiusi $\alpha \ne 0, \alpha \notin N$ bo'lganda 1 ga teng: $r = 1$.

(9) munosabatda $\alpha = -1$ deb olinsa, unda ushbu

$$\frac{1}{1+x} = \sum_{n=0}^{\infty}(-1)^n x^n = 1 - x + x^2 - x^3 + x^4 - \ldots + (-1)^n x^n + \ldots$$

formula hosil bo'ladi. Bu formulada x ni $-x$ ga almashtirib topamiz:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty}(-1)^n x^n = 1 + x + x^2 + \ldots + x^n + \ldots$$

1-misol. Ushbu

$$f(x) = \ln\frac{1+x}{1-x}$$

funksiya Teylor qatoriga yoyilsin.

Ma'lumki,

$$\ln\frac{1+x}{1-x} = \ln(1+x) - \ln(1-x)$$

bo'ladi.

Biz yuqorida

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{n-1}\frac{x^n}{n} + \ldots$$

$$\ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots \frac{x^n}{n} - \ldots$$

bo'lishini ko'rgan edik. Bu munosabatlardan foydalanib topamiz:

$$\ln(1+x) - \ln(1-x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{n-1}\frac{x^n}{n} + \ldots$$

$$-\left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \ldots - \frac{x^n}{n} - \ldots\right) = 2x + \frac{2x^3}{3} + \frac{2x^5}{5} + \ldots + \frac{2x^{2n-1}}{2n-1} + \ldots$$

Demak,

$$\ln\frac{1+x}{1-x}=2\left(x+\frac{x^3}{3}+\frac{x^5}{5}+...+\frac{x^{2n-1}}{2n-1}+...\right). \quad (10)$$

(10) darajali qatorning yaqinlashish radiusi $r=1$ bo'lib, yaqinlashish to'plamsi $(-1,1)$ bo'ladi.

2-misol. Ushbu

$$f(x)=\int_0^x\frac{\sin t}{t}dt$$

funksiya Teylor qatoriga yoyilsin.

Ma'lumki,

$$\sin t=t-\frac{t^3}{3!}+\frac{t^5}{5!}-...+(-1)^{n-1}\frac{t^{2n-1}}{(2n-1)!}+... .$$

Unda

$$\frac{\sin t}{t}=1-\frac{t^2}{3!}+\frac{t^4}{5!}-...+(-1)^{n-1}\frac{t^{2n-2}}{(2n-1)!}+...$$

bo'ladi. Bu darajali qatorni hadlab integrallab topamiz:

$$\int_0^x\frac{\sin t}{t}dt=\int_0^x\left(1-\frac{t^2}{3!}+\frac{t^4}{5!}-...+(-1)^{n-1}\frac{t^{2n-2}}{(2n-1)!}+...\right)dt=$$

$$=x-\frac{x^3}{3!\cdot 3}+\frac{x^5}{5!\cdot 5}-...+(-1)^{n-1}\frac{x^{2n-1}}{(2n-1)!\cdot(2n-1)}+...$$

Keyingi darajali qatorning yaqinlashish radiusi $r=+\infty$ bo'ladi.

3-misol. Ushbu

$$f(x)=\frac{2x-1}{x^2+x-6}$$

funksiya Teylor qatoriga yoyilsin va bu qatorning yaqinlashish radiusi topilsin.

Avvalo $f(x)$ funksiyani quyidagicha yozib olamiz:

$$f(x)=\frac{2x-1}{x^2+x-6}=\frac{1}{x+2}+\frac{1}{x-3}=\frac{1}{2\left(1+\frac{1}{2}x\right)}-\frac{1}{3\left(1-\frac{1}{3}x\right)}$$

Ma'lumki,

$$\frac{1}{1+x}=\sum_{n=0}^{\infty}(-1)^n\cdot x^n,$$

$$\frac{1}{1-x}=\sum_{n=0}^{\infty}x^n.$$

Bu formulalardan foydalanib topamiz:

$$\frac{1}{2\left(1+\frac{1}{2}x\right)}=\sum_{n=0}^{\infty}\frac{1}{2}(-1)^n\cdot\left(\frac{1}{2}x\right)^n=\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{n+1}}x^n \quad , (r=2)$$

$$\frac{1}{3\left(1-\frac{1}{3}x\right)}=\sum_{n=0}^{\infty}\frac{1}{3}\left(\frac{1}{3}x\right)^n=\sum_{n=0}^{\infty}\frac{1}{3^{n+1}}x^n \quad (r=3)$$

Demak,

$$\frac{2x-1}{x^2+x-6}=\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{n+1}}x^n-\sum_{n=0}^{\infty}\frac{1}{3^{n+1}}x^n=\sum_{n=0}^{\infty}\left(\frac{(-1)^n}{2^{n+1}}-\frac{1}{3^{n+1}}\right)x^n$$

bo’ladi.

Bu darajali qatorning yaqinlashish radiusi $r=2$ bo’ladi.

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