Funksional ketma-ketliklar va qatorlarning tekis yaqinlashishi. Koshi kriteriysi

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Annotatsiya: Ushbu maqolada Oliy matematikaning qiziqarli mavzularidan biri bo'lgan Funksional ketma-ketliklar va qatorlarning tekis yaqinlashishi hamda Koshi kriteriysi haqida ma'lumotlar keltirildi va quyidagi muammolar xal etildi. Funksional ketma-ketlik va limit funksiya tushunchalari. Funksional ketma-ketlikning tekis yaqinlashuvchiligi. Tekis yaqinlashuvchi funksional ketma-ketlikning xossalari. Funksional qatorning tekis yaqinlashuvchiligi. Bu hollarda qo'yilgan masalalarni yechishda quyida biz o'rganadigan qatorlar nazariyasi katta ahamiyatga ega.

Kalit so'zlar: Tekis yaqinlashuvchi funksional ketma-ketlikning xossalari. Funksional qatorning tekis yaqinlashuvchiligi, funksional ketma-ketlikning yaqinlashish to'plami.

Smooth approximation of functional sequences and series. Cauchy criterion

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Abstract: In this article, one of the interesting topics of Higher Mathematics, information about functional sequences and smooth convergence of series and the Cauchy criterion was presented, and the following problems were solved. Concepts of functional sequence and limit function. Smooth approximation of a functional sequence. Properties of linearly converging functional sequences. Smooth approximation of a functional series. The theory of series, which we will study below, is of great importance in solving the problems posed in these cases.

Keywords: Properties of a smooth converging functional sequence. Convergence of a functional series, set of convergence of a functional sequence.

1^0. Funksional ketma-ketlik va limit funkstiya tushunchalari. Aytaylik, har bir natural n songa $E \subset R$ to'plamda aniqlangan bitta $f_n(x)$ funkstiyani mos qo'yuvchi qoida berilgan bo'lsin. Bu qoidaga ko'ra

$$f_1(x),\ f_2(x)\ ,\ ...,\ f_n(x),... \qquad (1)$$

to'plam hosil bo'ladi. Uni funksional ketma-ketlik deyi-ladi. E to'plam (1) funksional ketma-ketlikning aniqlanish to'plami deyiladi.

Odatda, (1) funksional ketma-ketlik, uning n-hadi yordamida $\{f_n(x)\}$ yoki $f_n(x)$ kabi belgilanadi. Masalan,

$$f_n(x)=\frac{n+1}{n+x^2}\ :\frac{2}{1+x^2},\frac{3}{2+x^2},...,\frac{n+1}{n+x^2},...;$$

$$f_n(x)=\sin\frac{\sqrt{x}}{n}\ :\sin\frac{\sqrt{x}}{1},\sin\frac{\sqrt{x}}{2},...,\sin\frac{\sqrt{x}}{n},...$$

lar funksional ketma-ketliklar bo'ladi va ularning aniqlanish to'plami mos ravishda

$$E=R\ ,\ E=[0,+\infty)$$

bo'ladi. Ravshanki, x o'zgaruvchining biror tayinlangan $x=x_0\in E$ qiymatida ushbu

$$\{f_n(x_0)\}:\ f_1(x_0), f_2(x_0),..., f_n(x_0),...$$

sonlar ketma-ketligiga ega bo'lamiz.

1-ta'rif. Agar $\{f_n(x_0)\}$ sonli ketma-ketlik yaqinlashuvchi (uzoqlashuvchi) bo'lsa, $\{f_n(x)\}$ funksional ketma-ketlik $x=x_0$ nuqtada yaqinlashuvchi (uzoqlashuvchi) deyiladi. x_0 nuqta esa bu funksional ketma-ketlikning yaqinlashish (uzoqlashish) nuqtasi deyiladi.

2-ta'rif. $\{f_n(x)\}$ funksional ketma-ketlikning barcha yaqinlashish nuqtalarida iborat $E_0\subset E$ to'plam, $\{f_n(x)\}$ funksional ketma-ketlikning yaqinlashish to'plami deyiladi.

Masalan, ushbu

$$f_n(x)=x^n : x, x^2, x^3,..., x^n,...$$

funksional ketma-ketlik aniqlashish to'plami $E=R$ bo'lib, u $\forall x\in(-1,1]$ nuqtada yaqinlashuvchi, $x\in R\setminus(-1,1]$ da uzoqlashuvchi bo'ladi. Demak, ketma-ketlikning yaqinlashish to'plami $E_0=(-1,1]$ bo'ladi.

Faraz qilaylik, $\{f_n(x)\}$ funksional ketma-ketlikning yaqinlashish to'plami $E_0(E_0\subset R)$ bo'lsin. Ravshanki, bu holda har bir $x\in E_0$ da

$$f_1(x), f_2(x),..., f_n(x),...$$

ketma-ketlik yaqinlashuvchi, ya'ni

$$\lim_{n\to\infty} f_n(x)$$

mavjud bo'ladi. Endi har bir $x \in E$ ga $\lim_{n\to\infty} f_n(x)$ ni mos qo'ysak, ushbu

$$f : x \to \lim_{n\to\infty} f_n(x)$$

funkstiya hosil bo'ladi. Bu $f(x)$ funkstiya $\{f_n(x)\}$ funksional ketma-ketlikning limit funkstiyasi deyiladi:

$$\lim_{n\to\infty} f_n(x) = f(x) \qquad (x \in E_0).$$

Bu munosabat quyidagini anglatadi: ixtiyoriy $\varepsilon > 0$ son va har bir $x \in E_0$ uchun shunday natural $n_0 = n_0(\varepsilon, x)$ son topiladiki, ixtiyoriy $n > n_0$ da

$$|f_n(x) - f(x)| < \varepsilon,$$

ya'ni

$$\forall \varepsilon > 0 \;, \exists n_\circ = n_0(\varepsilon, x) \in N \;, \forall n > n_0 \; : |f_n(x) - f(x)| < \varepsilon$$

bo'ladi.

1-misol. Ushbu

$$f_n(x) = n \sin \frac{\sqrt{x}}{n}$$

funksional ketma-ketlikning limit funkstiyasi topilsin.

Berilgan funksional ketma-ketlik $E = [0, +\infty)$ da aniqlangan. Uning limit funkstiyasi

$$f(x) = \lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} n \sin \frac{\sqrt{x}}{n} = \lim_{n\to\infty} \frac{\sin \frac{\sqrt{x}}{n}}{\frac{\sqrt{x}}{n}} \cdot \sqrt{x} = \sqrt{x}$$

bo'ladi. Demak, funksional ketma-ketlik $E = [0, +\infty)$ da yaqinlashuvchi va

$$\lim_{n\to\infty} n \sin \frac{\sqrt{x}}{n} = \sqrt{x}.$$

2-misol. Ushbu

$$f_n(x) = x^n$$

funksional ketma-ketlikning limit funkstiyasi topilsin.

Bu funksional ketma-ketlik $E = R$ da aniqlangan. Ravshanki

$\forall x \in (1, +\infty)$ da $\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} x^n = +\infty$,

$\forall x \in (-1, 1)$ da $\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} x^n = 0$,

$x = 1$ da $\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} 1 = 1$,

$\forall x \in (-\infty, -1)$ da $\lim\limits_{n\to\infty} f_n(x)$ mavjud emas.

Demak, berilgan funksional ketma-ketlik $E_0 = (-1,1]$ yaqinla-shuvchi bo'lib, uning limit funkstiyasi

$$f(x) = \lim_{n\to\infty} x^n = \begin{cases} 0 \ , \ \text{агар} \ -1 < x < 1 \quad \text{бўлса}, \\ 1 \ , \ \text{агар} \quad x = 1 \quad \text{бўлса} \end{cases}$$

bo'ladi.

3-misol. Ushbu

$$f_n(x) = n^2\left(\sqrt[n]{x} - \sqrt[n+1]{x}\right) \qquad (x > 0)$$

funksional ketma-ketlikning limit funkstiyasi topilsin.

Berilgan funksional ketma-ketlikning limit funkstiyasi quyidagicha topiladi:

$$f(x) = \lim_{n\to\infty}(f_n(x)) = \lim_{n\to\infty} n^2\left(\sqrt[n]{x} - \sqrt[n+1]{x}\right) = \lim_{n\to\infty} n^2\left(x^{\frac{1}{n}} - x^{\frac{1}{n+1}}\right) =$$

$$= \lim_{n\to\infty} n^2 x^{\frac{1}{n+1}}\left(x^{\frac{1}{n}-\frac{1}{n+1}} - 1\right) = \lim_{n\to\infty} \frac{n^2}{n^2+n} x^{\frac{1}{n+1}} \cdot \frac{x^{\frac{1}{n^2+n}} - 1}{\frac{1}{n^2+n}} = \ln x.$$

2^0. *Funksional ketma-ketlikning tekis yaqinlashuv-chiligi.* Faraz qilaylik, $\{f_n(x)\}$

$$f_1(x)\ , \ f_2(x), ..., f_n(x), ...$$

funksional ketma-ketlik $E_\circ$ to'plamda yaqinlashuvchi (ya'ni yaqinlashish to'plami E_0) bo'lib, uning limit funkstiyasi $f(x)$ bo'lsin:

$$\lim_{n\to\infty} f_n(x) = f(x).$$

Ma'lumki, bu munosabat

$$\forall \varepsilon > 0 \ , \ \exists n_0 = n_0(\varepsilon, x) \in N, \ \forall n > n_0 : \ |f_n(x) - f(x)| < \varepsilon$$

bo'lishini anglatadi. Shuni ta'kidlash lozimki, yuqoridagi natural n_0 son ixtiyoriy olingan $\varepsilon > 0$ son bilan birga qaralayotgan $x \in E_0$ nuqtaga ham boђliq bo'ladi (chunki, $x \in E_0$ ning turli qiymatlarida ularga mos ketma-ketlik, umuman aytganda turlicha bo'ladi).

3-ta'rif. Agar $\forall \varepsilon > 0$ son olinganda ham shu $\varepsilon > 0$ gagina boђliq bo'lgan natural $n_0 = n_0(\varepsilon)$ son topilsaki, $\forall n > n_0$ va ixtiyoriy $x \in E_0$ da

$$|f_n(x) - f(x)| < \varepsilon$$

tengsizlik bajarilsa, ya'ni

$$\forall \varepsilon > 0 \ , \ \exists n_0 = n_0(\varepsilon) \in N \ , \ \forall n > n_0 \ , \ \forall x \in E_0 : \ |f_n(x) - f(x)| < \varepsilon$$

bo'lsa, $\{f_n(x)\}$ funksional ketma-ketlik E_0 to'plamda $f(x)$ ga tekis yaqinlashadi (funksional ketma-ketlik E_0 to'plamda tekis yaqinlashuvchi) deyiladi.

Shunday qilib, $\{f_n(x)\}$ funksional ketma-ketlik E_0 to'plamda $f(x)$ limit funkstiyaga ega bo'lsa, uning shu limit funkstiyasiga yaqinalishish ikki xil bo'lar ekan:

1) $\forall\varepsilon > 0$, $\exists n_0 = n_0(\varepsilon, x) \in N$ $,\forall n > n_0$: $|f_n(x) - f(x)| < \varepsilon$

bo'lsa, $\{f_n(x)\}$ funksional ketma-ketlik E_0 da $f(x)$ ga yaqinlashadi (oddiy yaqinlashadi). Bu holda

$$f_n(x) \to f(x) \qquad (x \in E_0)$$

kabi belgilanadi.

2) $\forall\varepsilon > 0$, $\exists n_0 = n_0(\varepsilon) \in N$, $\forall n > n_0$, $\forall x \in E_0$: $|f_n(x) - f(x)| < \varepsilon$

bo'lsa, $\{f_n(x)\}$ funksional ketma-ketlik E_0 da $f(x)$ ga tekis yaqinlashadi. Bu holda

$$f_n(x) \rightrightarrows f(x) \qquad (x \in E_0)$$

kabi belgilanadi.

Ravshanki, $\{f_n(x)\}$ funksional ketma-ketlik E_0 to'plamda $f(x)$ funkstiyaga tekis yaqinlashsa u shu to'plamda $f(x)$ ga yaqinlashadi:

$$f_n(x) \rightrightarrows f(x) \;\Rightarrow\; f_n(x) \to f(x) \qquad (x \in E_0).$$

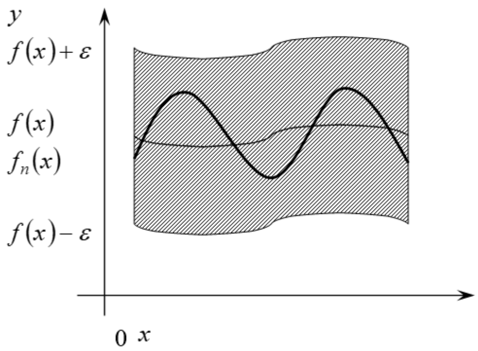
Aytaylik,

$$f_n(x) \rightrightarrows f(x) \qquad (x \in E_0)$$

bo'lsin . Bu holda $\forall n > n_0$ va $\forall x \in E_0$ da

$$|f_n(x) - f(x)| < \varepsilon \text{, ya'ni } f(x) - \varepsilon < f_n(x) < f(x) + \varepsilon$$

bo'ladi. Bu esa $\{f_n(x)\}$ funksional ketama-ketlikning biror hadidan boshlab, keyingi barcha hadlari $f(x)$ funkstiyaning "ε-oraliђi"da butunlay joylashishini bildiradi (29-chizma)



29-chizma

4-misol. Ushbu

$$f_n(x)=\frac{\sin nx}{n}$$

funksional ketma-ketlikning R da tekis yaqinlashuvchiligi ko'rsatilsin.

Ravshanki,

$$\lim_{n\to\infty} f_n(x)=\lim_{n\to\infty}\frac{\sin nx}{n}=0.$$

Demak, limit funkstiya $f(x)=0$.

Agar $\forall\varepsilon>0$ son olinganda $n_0=\left[\frac{1}{\varepsilon}\right]$ deyilsa, unda $\forall n>n_0$ va $\forall x\in R$ uchun

$$|f_n(x)-f(x)|=\left|\frac{\sin nx}{n}-0\right|=\left|\frac{\sin nx}{n}\right|\le\frac{1}{n}\le\frac{1}{n_0+1}<\varepsilon$$

bo'lishini topamiz. Demak ta'rifga binoan

$$\frac{\sin nx}{n}\rightrightarrows 0$$

bo'ladi.

Faraz qilaylik, $\{f_n(x)\}$ funksional ketma-ketlik E_0 to'plamda $f(x)$ limit funkstiyaga ega bo'lsin.

1-teorema. $\{f_n(x)\}$ funksional ketma-ketlik E_0 to'plamda $f(x)$ funkstiyaga tekis yaqilashishi uchun

$$\lim_{n\to\infty}\sup_{x\in E_0}|f_n(x)-f(x)|=0$$

bo'lishi zarur va etarli.

Zarurligi. Aytaylik,

$$f_n(x)\rightrightarrows f(x) \qquad (x\in E_0)$$

bo'lsin. Ta'rifga binoan

$$\forall\varepsilon>0,\ \exists n_0=n_0(\varepsilon)\in N,\ \forall n>n_0,\ \forall x\in E_0\ :\ |f_n(x)-f(x)|<\varepsilon$$

bo'ladi. Bu tengsizlikdan

$$\sup_{x\in E_0}|f_n(x)-f(x)|\le\varepsilon$$

bo'lib, undan

$$\lim_{n\to\infty}\sup_{x\in E_0}|f_n(x)-f(x)|=0$$

bo'lishi kelib chiqadi.

Etarliligi. Aytaylik

$$\lim_{n\to\infty}\sup_{x\in E_0}|f_n(x)-f(x)|=0$$

bo'lsin. Limit ta'rifga ko'ra

$$\forall \varepsilon > 0,\ \exists n_0 \in N \quad \forall n > n_0,: \sup_{x\in E_0} |f_n(x) - f(x)| < \varepsilon$$

bo'ladi. Ravshanki

$$|f_n(x) - f(x)| \le \sup_{x\in E_0} |f_n(x) - f(x)|.$$

U holda $\forall x \in E_0$ uchun

$$|f_n(x) - f(x)| < \varepsilon$$

bo'ladi. Bundan

$$f_n(x) \rightrightarrows f(x) \qquad (x \in E_0)$$

bo'lishi kelib chiqadi.

5-misol. Ushbu

$$f_n(x) = \sqrt{x^2 + \frac{1}{n^2}}$$

funksional ketama-ketlikning $E_0 = R$ da tekis yaqinlashuv-chiligi ko'rsatilsin. Berilgan funksional ketma-ketlikning limit funkstiyasi

$$f(x) = \lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} \sqrt{x^2 + \frac{1}{n^2}} = |x| \qquad (x \in R)$$

bo'ladi. Endi

$$\sup_x |f_n(x) - f(x)|$$

ni topamiz:

$$\sup_{x\in R} \left| \sqrt{x^2 + \frac{1}{n^2}} - |x| \right| = \sup_{x\in R} \left| \frac{\frac{1}{n^2}}{\left(\sqrt{x^2 + \frac{1}{n^2}} + |x|\right)} \right| = \sup_{x\in R} \frac{1}{n^2} \cdot \frac{1}{\left(\sqrt{x^2 + \frac{1}{n^2}} + |x|\right)} = \frac{1}{n}$$

Demak,

$$\lim_{n\to\infty} \sup_{x\in R} \left| \sqrt{x^2 + \frac{1}{n^2}} - |x| \right| = \lim_{n\to\infty} \frac{1}{n} = 0$$

bo'lib,

$$\sqrt{x^2 + \frac{1}{n^2}} \rightrightarrows |x| \qquad (x \in R)$$

bo'ladi.

Eslatma. Agar $\{f_n(x)\}$ funksional ketma-ketligi uchun $E \subset R$ to'plamda

$$\lim_{n\to\infty} \sup_{x\in E} |f_n(x) - f(x)| \ne 0$$

bo'lsa, $\{f_n(x)\}$ funksional ketma-ketlik E da tekis yaniqla-shishi shart emas.

Endi funksional ketma-ketlikning limit funkstiyaga ega bo'lishi va unga tekis yaђinlashishini ifodalovchi teoremani keltiramiz:

2-teorema (Koshi teoremasi). $\{f_n(x)\}$ funksional ketma-ketlik E to'plamda limit funkstiyaga ega bo'lishi va unga tekis yaqinlashishi uchun $\forall \varepsilon > 0$ son olinganda ham shunday $n_0 = n_{0(\varepsilon)} \in N$ topilib, $\forall n > n_0$, $\forall p \in N$ va $\forall x \in E$ da

$$\left| f_{n+p}(x) - f_n(x) \right| < \varepsilon,$$

ya'ni

$$\forall \varepsilon > 0\ , \exists n_0 = n_0(\varepsilon) \in N,\ \forall n > n_0,\ \forall p \in N \text{ va } \forall x \in E \text{ da}$$

$$\left| f_{n+p}(x) - f_n(x) \right| < \varepsilon \qquad (2)$$

bo'lishi zarur va etarli.

Zarurligi. Aytaylik, E to'plamda $\{f_n(x)\}$ funksional ketma-ketlik limit funkstiya $f(x)$ ga ega bo'lib, unga tekis yaqinlashsin:

$$f_n(x) \rightrightarrows f(x). \qquad (x \in E_0)$$

Tekis yaqinlashish ta'rifiga ko'ra

$\forall \varepsilon > 0,\ \exists n_0 = n_0(\varepsilon) \in N,\ \forall k > n_0,\ \forall x \in E : \left| f_k(x) - f(x) \right| < \dfrac{\varepsilon}{2}$ bo'ladi. Xususan, $k = n,\ \ n > n_0$ va $k = n + p,\ p \in N$ da

$$\left| f_n(x) - f(x) \right| < \frac{\varepsilon}{2}, \quad \left| f_{n+p}(x) - f(x) \right| < \frac{\varepsilon}{2}$$

tengsizliklar bajarilib, ulardan

$$\left| f_{n+p}(x) - f_n(x) \right| = \left| f_{n+p}(x) - f(x) - (f_n(x) - f(x)) \right| \le$$

$$\le \left| f_{n+p}(x) - f(x) \right| + \left| f_n(x) - (x) \right| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

bo'lishi kelib chiqadi. Demak, (2) shart o'rinli.

Etarliligi. $\{f_n(x)\}$ funksional ketma-ketlik uchun (2) shart bajarilsin. Uni quyidagicha yozamiz:

$\forall \varepsilon > 0,\ \exists n_0 = n_0(\varepsilon) \in N,\ \forall n > n_0,\ \forall p \in N, \forall x \in E$ da

$$\left| f_{n+p}(x) - f_n(x) \right| < \frac{\varepsilon}{2} \qquad (3)$$

bo'ladi.

Ravshanki, tayin $x_0 \in E$ da $\{f_n(x_0)\}$ sonlar ketma-ketligi uchun (3) shartning bajarilishidan uning fundamental ketma-ketlik ekanligi kelib chiqadi. Koshi teoremasiga ko'ra $\{f_n(x_0)\}$ yaqinlashuvchi bo'ladi. Binobarin, chekli

$$\lim_{n\to\infty} f_n(x_0) \qquad (4)$$

limit mavjud.

Modomiki, har bir $x \in E$ da (4) limit mavjud bo'lar ekan, unda avval ayganimizdek, E to'plamda aniqlangan

$$x \to \lim_{n\to\infty} f_n(x) \qquad (x \in E)$$

funkstiya hosil bo'ladi Uni $f(x)$ bilan belgilaymiz. Bu funkstiya $\{f_n(x)\}$ funksional ketma-ketlikning limit funkstiyasi bo'ladi:

$$f_n(x) \to f(x) \qquad (x \in E).$$

Endi (3) tengsizlikda, n va x larni tayinlab $(n > n_0,\ x \in E)$ $p \to \infty$ da limitga o'tamiz. Natijada

$$|f(x) - f_n(x)| \le \frac{\varepsilon}{2} < \varepsilon$$

hosil bo'ladi. Bu

$$f_n(x) \rightrightarrows f(x) \qquad (x \in E_0)$$

bo'lishini bildiradi.

6-misol. Ushbu

$$f_n(x) = \frac{\ln nx}{\sqrt{nx}}$$

funksional ketma-ketlik $E = (0,1)$ to'plamda tekis yaqinlashuv-chilikka tekshirilsin.

Agar ixtiyoriy $k \in N$ uchun

$$n = k \ , \ p = k = n \ , x^* = \frac{1}{k} = \frac{1}{n}$$

deyilsa,

$$|f_{n+p}(x) - f(x)| = \left| f_{2n}\left(\frac{1}{n}\right) - f_n\left(\frac{1}{n}\right) \right| = \left| \frac{\ln 2}{\sqrt{2}} - \ln 1 \right| = \frac{\ln 2}{\sqrt{2}} = \varepsilon_0$$

bo'ladi. Demak,

$$\exists \varepsilon_0 = \frac{\ln 2}{\sqrt{2}} \ \forall k \in N, \exists n \ge k, \exists p \in N, \ \exists x^* = \frac{1}{n} \in (0,1) : \left| f_{n+p}(x^*) - f_n(x^*) \right| \ge \varepsilon_0 .$$

Bu esa yuqoridagi teoremaning shartini bajarilmasligini ko'rsatadi. Demak, berilgan funksional ketma-ketlik $E = (0,1)$ da tekis yaqinlashuvchi emas.

Aytaylik, $\{f_n(x)\}$ funksional ketma-ketlik E to'plamda yaqinlashuvchi bo'lib, $f(x)$ funkstiya uning limit funkstiyasi bo'lsin:

$$f_n(x) \to f(x) \qquad (x \in E).$$

Agar

$$\exists \varepsilon_0 > 0 \text{ , } \forall k \in N \text{ , } \exists n > k \text{ , } \exists x^* \in E : \left| f_n(x^*) - f(x^*) \right| \geq \varepsilon_0$$

bo'lsa, $\{f_n(x)\}$ funksional ketma-ketlik E to'plamda $f(x)$ funkstiyaga notekis yaqinlashadi deyiladi.

7-misol. Ushbu

$$f_n(x) = n \sin \frac{1}{nx}$$

funksional ketma-ketlik $E = (0,1)$ da tekis yaqinlashishiga tekshirilsin.

Ravshanki,

$$\lim_{n\to\infty} f_n(x) = \lim_{n\to\infty} n \sin \frac{1}{nx} = \lim_{n\to\infty} \frac{\sin \frac{1}{nx}}{\frac{1}{nx} \cdot x} = \frac{1}{x}.$$

Demak, berilgan funksional ketma-ketlikning limit funkstiyasi $f(x) = \frac{1}{x}$ bo'ladi.

Aytaylik, $x^* = \frac{1}{n}$ bo'lsin. Unda

$$\left| f_n(x^*) - f(x^*) \right| = |n \sin 1 - n| \geq 1 - \sin 1 = \varepsilon_0$$

munosabat ixtiyoriy $n \in N$ da o'rinli bo'ladi.

Demak, $f_n(x) = n \sin \frac{1}{nx}$ funksional ketma-ketlik limit funkstiya $f(x) = \frac{1}{x}$ ga $E = (0,1)$ da tekis yaqinlashmaydi.

3^0. Tekis yaqinlashuvchi funksional ketma-ketlikning xossalari. Tekis yaqinlashuvchi funkstiyaonal ketma-ketliklar qator xossalarga ega. Bu xossalarni keltiramiz.

Aytaylik. $\{f_n(x)\}$:

$$f_1(x) \text{ , } f_2(x), \ldots, f_n(x), \ldots$$

funksional ketma-ketlik $E \subset R$ to'plamda yaqinlashuvchi bo'lib, $f(x)$ uning limit funkstiyasi bo'lsin:

$$f_n(x) \to f(x) \qquad (x \in E).$$

1-xossa. Agar $\{f_n(x)\}$ funksional ketma-ketlikning har bir $f_n(x)$ $(n=1,2,3,...)$ hadi E to'plamda uzluksiz bo'lib,

$$f_n(x) \rightrightarrows f(x) \qquad (x \in E)$$

bo'lsa, limit funkstiya $f(x)$ shu E to'plamda uzluksiz bo'ladi.

Demak, bu holda

$$\lim_{t\to x}\left(\lim_{n\to\infty} f_n(t)\right) = \lim_{n\to\infty}\left(\lim_{t\to x} f_n(t)\right)$$

munosabat o'rinli bo'ladi.

2-xossa. Agar $\{f_n(x)\}$ funksional ketma-ketlikning har bir $f_n(x)$ $(n=1,2,3,...)$ hadi $E=[a,b]$ da uzluksiz bo'lib,

$$f_n(x) \rightrightarrows f(x) \qquad (x \in [a,b])$$

bo'lsa,

$$\lim_{n\to\infty}\int_a^b f_n(x)dx = \int_a^b f(x)dx$$

bo'ladi.

Demak, bu holda

$$\lim_{n\to\infty}\int_a^b f_n(x)dx = \int_a^b \left(\lim_{n\to\infty} f_n(x)\right)dx$$

munosabat o'rinli bo'ladi.

3-xossa. Agar $\{f_n(x)\}$ funksional ketma-ketlikning har bir $f_n(x)$ $(n=1,2,3,...)$ hadi $E=[a,b]$ da uzluksiz $f_n'(x)$ $(n=1,2,3,...)$ hosilalarga ega bo'lib,

$$f_n'(x) \rightrightarrows \varphi(x) \qquad (x \in [a,b])$$

bo'lsa,

$$\varphi(x) = f'(x)$$

bo'ladi.

Shu kabi xossalarga keyinroq o'rganiladigan tekis yaqinlashuvchi funksional qatorlar ham ega bo'ladi. Ayni paytda, ular bir mulohaza asosida isbotlanadi Mazkur xossalarning isbotini funksional qatorlarga nisbatan keltiramiz.

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