Furye qatori. Funksiyalarni Furye qatoriga yoyish

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Annotatsiya: Ushbu maqolada matematikaning eng muhim mavzularidan biri bo'lgan Furye qatori. Funksiyani Furye qatoriga yoyish tog'risida malumot keltirildi va mavjud muanmolar xal etildi. Agar f (x) funksiya [a;b] kesmada monoton bo'lsa yoki [a;b] kesmani chekli sondagi qismiy kesmalarga bo'lish mumkin bo'lsa va bu kesmalarning har birida f (x) funksiya monoton (faqat o'ssa yoki faqat kamaysa) yoki o'zgarmas bo'lsa, f (x) funksiyaga [a;b] kesmada bo'laklimonoton funksiya deyiladi. Agar f (x) funksiya [a;b] kesmada chekli sondagi birinchi tur uzilish nuqtalariga ega bo'lsa, f (x) funksiyaga [a;b] kesmada bo'lakli-uzluksiz funksiya deyiladi. Agar f (x) funksiya [a;b] kesmada uzluksiz yoki bo'lakli-uzluksiz bo'lib, bo'lakli-monoton bo'lsa f (x) funksiya [a;b] kesmada Dirixle shartlarini qanoatlantiradi deyiladi. Bu hollarda qo'yilgan masalalarni yechishda quyida biz o'rganadigan qatorlar nazariyasi katta ahamiyatga ega.

Kalit so'zlar: Furye qatori, Furye koeffitsiyentlari. Funksiyalarni Furye qatoriga yoyish.

Fourier series. Fourier series expansion of functions

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Abstract: In this article, the Fourier series is one of the most important topics in mathematics. Information on the expansion of the function into the Fourier series was given and the existing problems were solved. If the function f (x) is monotone in the section [a;b] or if the section [a;b] can be divided into a finite number of partial sections, and in each of these sections the function f (x) is monotone (only if or only decreases) or is constant, the function f (x) is called a piecewise monotone function on the cross section [a;b]. If the function f (x) has a finite number of discontinuities of the first type on the section [a;b], then the function f (x) is called a piecewise-continuous function on the section [a;b]. If the function f (x) is continuous or piecewise-continuous in the cross section [a;b], and is piecewise monotone, then the function f (x) is said to satisfy the Dirichlet conditions in the cross section [a;b]. The

theory of series, which we will study below, is of great importance in solving the problems posed in these cases.

Keywords: Fourier series, Fourier coefficients. Fourier series expansion of functions.

1. Furye qatori.

Faraz qilaylik, $f(x)$ funksiya $R=(-\infty,+\infty)$ da berilgan bo‘lsin. Ma’lumki, shunday $T\in R\setminus\{0\}$ son topilsaki, $\forall x\in R$ da

$$f(x+T)=f(x)$$

tenglik bajarilsa, $f(x)$ davriy funksiya, $T\neq 0$ son esa uning davri deyiladi.

Agar $T\neq 0$ son $f(x)$ funksiyaning davri bo‘lsa, u holda

$$kT \quad (k=\pm 1,\pm 2,\ldots)$$

sonlar ham shu funksiyaning davri bo‘ladi.

Agar $f(x)$ va $g(x)$ davriy funksiyalar bo‘lib, $T\neq 0$ ularning davri bo‘lsa,

$$f(x)\pm g(x),\ \ f(x)\cdot g(x),\ \ \frac{f(x)}{g(x)} \quad (g(x)\neq 0)$$

funksiyalar ham davriy bo‘lib, ularning davri T ga teng bo‘ladi.

$y=\sin x,\ y=\cos x$ funksiyalar $T=2\pi$ davrli funksiya bo‘lgan holda ushbu

$$\varphi(x)=a\cos\alpha x+b\sin\alpha x \quad (a,b,\alpha - \text{ o‘zgarmas, } \alpha\neq 0)$$

funksiya ham davriy funksiya bo‘lib, uning davri $T=\dfrac{2\pi}{\alpha}$ bo‘ladi. Haqiqatan ham,

$$\varphi\left(x+\frac{2\pi}{\alpha}\right)=a\cos\left[\alpha\left(x+\frac{2\pi}{\alpha}\right)\right]+b\sin\left[\alpha\left(x+\frac{2\pi}{\alpha}\right)\right]=$$
$$=a\cos(\alpha x+2\pi)+b\sin(\alpha x+2\pi)=a\cos\alpha x+b\sin\alpha x=\varphi(x)$$

bo‘ladi.

Bu $\varphi(x)=a\cos\alpha x+b\sin\alpha x$ sodda davriy funksiya bo‘lib, u garmonika deb ataladi.

Aytaylik, $f(x)$ funksiya $[-\pi,\pi]$ da uzluksiz bo‘lsin. Unda

$$f(x)\cos nx,\ f(x)\sin nx \quad (n=1,2,3,\ldots)$$

funksiyalar ham $[-\pi,\pi]$ da uzluksiz bo‘lib, ular $[-\pi,\pi]$ da integrallanuvchi bo‘ladi. Bu integrallarni quyidagicha belgilaymiz:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx,$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nxdx, \quad (n=1,2,\dots)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nxdx. \quad (n=1,2,\dots) \qquad (1)$$

Bu sonlardan foydalanib, ushbu

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx + b_n \sin nx) \qquad (2)$$

qatorni (uni trigonometrik qator deyiladi) hosil qilamiz.

(2) qator funksional qator bo‘lib, uning har bir hadi garmonikadan iborat.

Ta’rif. (2) funksional qator $f(x)$ funksiyaning Furye qatori deyiladi. (1) munosabatlar bilan aniqlangan

$$a_0, a_1, b_1, a_2, b_2, \dots, a_n, b_n, \dots$$

sonlar Furye koeffitsiyentlari deyiladi.

7.2. Funksiyalarni Furye qatoriga yoyish.

Demak, berilgan $f(x)$ funksiyaning Furye koeffitsiyentlari shu funksiyaga bog‘liq bo‘lib, (2) formulalar yordamida aniqlanadi, qator esa quyidagicha:

$$f(x) \sim \frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx + b_n \sin nx).$$

belgilanadi.

1-misol. Ushbu $f(x)=e^{\alpha x}$ $(-\pi \le x \le \pi\ , \alpha \ne 0)$ funksiyaning Furye qatori topilsin.

(1) formulalardan foydalanib, berilgan funksiyaning Furye koeffitsiyentlarini hisoblaymiz:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{\alpha x}dx = \frac{1}{\alpha\pi}\left(e^{\alpha\pi}-e^{-\alpha\pi}\right) = \frac{2}{\alpha\pi}sh\alpha\pi,$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{\alpha x}\cos nxdx = \frac{1}{\pi}\frac{\alpha\cos nx + n\sin nx}{\alpha^2+n^2}e^{\alpha x}\Big|_{-\pi}^{\pi} =$$

$$=(-1)^n\frac{1}{\pi}\cdot\frac{2\alpha}{\alpha^2+n^2}sh\alpha\pi \qquad (n=1,2,\dots),$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{\alpha x}\sin nxdx = \frac{1}{\pi}\frac{\alpha\sin nx - n\cos nx}{\alpha^2+n^2}e^{\alpha x}\Big|_{-\pi}^{\pi} =$$

$$=(-1)^{n-1}\frac{1}{\pi}\cdot\frac{2n}{\alpha^2+n^2}sh\alpha\pi \qquad (n=1,2,\dots).$$

Demak,

$$f(x)=e^{\alpha x}$$

funksiyaning Furye qatori

$$f(x)=e^{\alpha x}\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)=$$
$$=\frac{2sh\alpha\pi}{\pi}\left[\frac{1}{2\alpha}+\sum_{n=1}^{\infty}\frac{(-1)^n}{\alpha^2+n^2}(\alpha\cos nx-n\sin nx)\right]$$

bo‘ladi.

Aytaylik, $f(x)$ funksiya $[-\pi,\pi]$ da berilgan juft funksiya bo‘lsin: $f(-x)=f(x)$. U holda

$f(x)\cdot\cos nx$ juft, $f(x)\cdot\sin nx$ toq $(n=1,2,3,...)$ funksiya bo‘ladi.

(1) formulalardan foydalanib, $f(x)$ funksiyaning Furye koeffitsiyentlarini topamiz:

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx=\frac{1}{\pi}\left[\int_{-\pi}^{0}f(x)\cos nxdx+\int_{0}^{\pi}f(x)\cos nxdx\right]=$$
$$=\frac{1}{\pi}\left[\int_{0}^{\pi}f(x)\cos nxdx+\int_{0}^{\pi}f(x)\cos nxdx\right]=$$
$$\frac{2}{\pi}\int_{0}^{\pi}f(x)\cos nxdx \quad (n=0,1,2,\ldots).$$
$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx=\frac{1}{\pi}\left[\int_{-\pi}^{0}f(x)\sin nxdx+\int_{0}^{\pi}f(x)\sin nxdx\right]=$$
$$=\frac{1}{\pi}\left[-\int_{0}^{\pi}f(x)\sin nxdx+\int_{0}^{\pi}f(x)\sin nxdx\right]=0 \qquad (n=1,2,\ldots).$$

Demak, juft $f(x)$ funksiyaning Furye koeffitsiyentlari

$$a_n=\frac{2}{\pi}\int_{0}^{\pi}f(x)\cos nxdx \quad (n=0,1,2,\ldots)$$
$$b_n=0 \quad (n=1,2,\ldots)$$

bo‘lib, Furye qatori

$$f(x)\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos nx$$

bo‘ladi.

Aytaylik, $f(x)$ funksiya $[-\pi,\pi]$ da berilgan toq funksiya bo‘lsin:
$f(-x)=-f(x)$. U holda

$f(x)\cdot\cos nx$ toq, $f(x)\cdot\sin nx$ juft $(n=1,2,3,...)$ funksiya bo‘ladi.

(1) formulalardan foydalanib, $f(x)$ funksiyaning Furye koeffitsiyentlarini topamiz:

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nxdx=\frac{1}{\pi}\left[\int_{-\pi}^{0}f(x)\cos nxdx+\int_{0}^{\pi}f(x)\cos nxdx\right]=$$
$$=\frac{1}{\pi}[-\int_{0}^{\pi}f(x)\cos nxdx+f(x)\cos nxdx]=0 \qquad (n=0,1,2,\ldots),$$
$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nxdx=\frac{1}{\pi}\left[\int_{-\pi}^{0}f(x)\sin nxdx+\int_{0}^{\pi}f(x)\sin nxdx\right]=$$
$$=\frac{2}{\pi}\left[\int_{0}^{\pi}f(x)\sin nxdx\right] \qquad (n=1,2,\ldots).$$

Demak, toq $f(x)$ funksiyaning Furye koeffitsiyentlari

$$a_n=0, \quad (n=0,1,2,\ldots),$$
$$b_n=\frac{2}{\pi}\int_{0}^{\pi}f(x)\sin nxdx, \ (n=1,2,\ldots)$$

bo‘lib, Furye qatori

$$f(x)\sim\sum_{n=1}^{\infty}b_n\sin nx$$

bo‘ladi.

2-misol. Ushbu $f(x)=x^2 \quad (-\pi\le x\le\pi)$ juft funksiyaning Furye qatori topilsin. Avvalo berilgan funksiyaning Furye koeffitsiyentlarini topamiz:

$$a_0=\frac{2}{\pi}\int_{0}^{\pi}x^2dx=\frac{2}{3}\pi^2,$$
$$a_n=\frac{2}{\pi}\int_{0}^{\pi}x^2\cos nxdx=\frac{2}{\pi}x^2\frac{\sin nx}{n}\bigg|_0^{\pi}-\frac{4}{n\pi}\int_{0}^{\pi}x\sin nxdx=$$
$$=\frac{4}{\pi n}\left(\frac{x\cos nx}{n}\bigg|_0^{\pi}-\frac{1}{n}\int_{0}^{\pi}\cos nxdx\right)=(-1)^n\cdot\frac{4}{n^2}.\ (n=1,2,\ldots)$$

Demak, $f(x)=x^2$ funksiyaning Furye qatori

$$f(x)=x^2\sim\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}(-1)^n\frac{\cos nx}{n^2}$$

bo‘ladi.

3-misol. Ushbu $f(x)=x \quad (-\pi\le x\le\pi)$ toq funksiyaning Furye qatori topilsin. Berilgan funksiyaning Furye koeffitsiyentlarini hisoblaymiz:

$$b_n=\frac{2}{\pi}\int_{0}^{\pi}x\sin nxdx=\frac{2}{\pi}\left(-\frac{x\cos nx}{n}\bigg|_0^{\pi}+\frac{1}{n}\int_{0}^{\pi}\cos nxdx\right)=\frac{2(-1)^{n-1}}{n}.$$

Demak, $f(x)=x$ funksiyaning Furye qatori

$$f(x)\sim\sum_{n=1}^{\infty}(-1)^{n-1}\frac{2}{n}\sin nx$$

bo‘ladi.

Faraz qilaylik, $f(x)$ funksiya $[-p,p]$ $(p>0)$ segmentda uzluksiz bo‘lsin. Ma’lumki, ushbu

$$t=\frac{\pi}{p}x$$

almashtirish $[-p,p]$ oraliqni $[-\pi,\pi]$ ga o‘tkazadi, ya’ni x o‘zgaruvchi $[-p,p]$ da o‘zgarganda t o‘zgaruvchi $[-\pi,\pi]$ da o‘zgaradi. Endi

$$f(x)=f\left(\frac{p}{\pi}t\right)=\varphi(t).$$

deymiz. Unda $\varphi(t)$ funksiya $[-\pi,\pi]$ oraliqda berilgan uzluksiz funksiya bo‘ladi. Bu funksiyaning Furye koeffitsiyentlari

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\varphi(t)\cos ntdt,\ (n=0,1,2,\ldots)$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\varphi(t)\sin ntdt\quad (n=1,2,\ldots)$$

ni topib, Furye qatorini yozamiz:

$$\varphi(t)\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos nt+b_n\sin nt\right).$$

Modomiki,

$$t=\frac{\pi}{p}x$$

ekan, unda

$$\varphi\left(\frac{\pi}{p}x\right)\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos n\frac{\pi}{p}x+b_n\sin n\frac{\pi}{p}x\right),$$

bo‘lib, uning koeffitsiyentlari

$$a_n=\frac{1}{p}\int_{-p}^{p}\varphi\left(\frac{\pi}{p}x\right)\cos n\frac{\pi}{p}xdx,\quad (n=0,1,2\ldots)$$

$$b_n=\frac{1}{p}\int_{-p}^{p}\varphi\left(\frac{\pi}{p}x\right)\sin n\frac{\pi}{p}xdx.\quad (n=1,2\ldots)$$

bo‘ladi. Natijada $[-p,p]$ da berilgan $f(x)$ funksiyaning Furye qatorini quyidagicha

$$f(x)\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{p}+b_n\sin\frac{n\pi x}{p}\right)$$

bo‘lishini topamiz, bunda

$$a_n = \frac{1}{p}\int_{-p}^{p} f(x)\cos\frac{n\pi x}{p}dx \quad (n = 0,1,2\ldots)$$

$$b_n = \frac{1}{p}\int_{-p}^{p} f(x)\sin\frac{n\pi}{p}xdx \quad (n = 1,2\ldots)$$

4-misol. Ushbu $f(x) = e^x \quad (-1 \le x \le 1)$ funksiyaning Furye qatori topilsin.

Yuqoridagi formulalardan foydalanib, $f(x) = e^x$ funksiyaning Furye koeffitsiyentilarini topamiz:

$$a_0 = \int_{-1}^{1} e^x dx = e - e^{-1},\ a_n = \int_{-1}^{1} e^x \cos n\pi x dx = \frac{n\pi\sin n\pi x - \cos n\pi x}{1+n^2\pi^2}e^x\Big|_{-1}^{1} =$$

$$= \frac{1}{1+n^2\pi^2}\left(e\cos n\pi - e^{-1}\cos n\pi\right) = (-1)^n\frac{e-e^{-1}}{1+n^2\pi^2} \quad (n = 1,2,\ldots),$$

$$b_n = \int_{-1}^{1} e^x \cos n\pi x dx = \frac{\sin n\pi x - n\pi\cos n\pi x}{1+n^2\pi^2}e^x\Big|_{-1}^{1} =$$

$$= \frac{1}{1+n^2\pi^2}\left(en\pi\cos n\pi + n\pi e^{-1}\cos n\pi\right) =$$

$$= \frac{n\pi(-1)^n}{1+n^2\pi^2}\left(e^{-1} - e\right) = (-1)^{n+1}\frac{e-e^{-1}}{1+n^2\pi^2} \quad (n = 1,2,\ldots)$$

Demak,

$$f(x) = e^x \quad (-1 \le x \le 1)$$

funksiyaning Furye qatori

$$e^x \sim \frac{e-e^{-1}}{2} + \left(e - e^{-1}\right)\sum_{n=1}^{\infty}\left[\frac{(-1)^n}{1+n^2\pi^2}\cos n\pi + \frac{(-1)^{n+1}}{1+n^2\pi^2}n\pi\sin n\pi x\right]$$

bo‘ladi.

Aytaylik, $f(x)$ funksiya $[a,b]$ da berilgan bo’lsin. $[a,b]$ segment a_k nuqtalar yordamida bo‘laklarga ajratilgan. $(a_0 = a,\ a_n = b)$.

Agar har bir (a_k, a_{k+1}) $(k = 0,1,2,\ldots,n-1)$ da $f(x)$ funksiya differensiallanuvchi bo‘lib, $x = a_k$ nuqtalarda chekli o‘ng

$$f'(a_k + 0) \quad (k = 0,1,2,\ldots,n-1),$$

va chap

$$f'(a_k - 0) \quad (k = 0,1,2,\ldots,n)$$

hosilalarga ega bo‘lsa, $f(x)$ funksiya $[a,b]$ da bo‘lakli-differensiallanuvchi deyiladi.

Endi Furye qatorining yaqinlashuvchi bo‘lishi haqidagi teoremani isbotsiz keltiramiz.

Teorema. 2π davrli $f(x)$ funksiya $[-\pi,\pi]$ oraliqda bo‘lakli-differensiallanuvchi bo‘lsa, u holda bu funksiyaning Furye qatori

$$f(x)\sim\frac{a_0}{2}+\sum_{k=1}^{\infty}\left(a_k\cos kx+b_k\sin kx\right)$$

$[-\pi,\pi]$ da yaqinlashuvchi bo‘lib, uning yig‘indisi

$$\frac{f(x+0)+f(x-0)}{2}$$

ga teng bo‘ladi.

5-misol. Ushbu $f(x)=\cos ax\quad(-\pi\le x\le\pi\ ,\ a\ne n\in Z)$ funksiyaning Furye qatori topilsin va u yaqinlashishga tekshirilsin.

Bu funksiyaning Furye koeffitsiyentlarini topamiz. Qaralayotgan funksiya juft bo‘lgani uchun

$$b_n=0\quad(n=1,2,3,\ldots)$$

bo‘lib,

$$a_n=\frac{2}{\pi}\int_0^{\pi}\cos ax\cos nxdx=\int_0^{\pi}\left[\cos(a-n)x+\cos(a+n)x\right]dx=$$
$$=\frac{\sin a\pi}{\pi}(-1)^n\left[\frac{1}{a+n}+\frac{1}{a-n}\right]$$

bo‘ladi. Demak,

$$f(x)\sim\frac{\sin a\pi}{\pi}\left[\frac{1}{a}+\sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{a+n}+\frac{1}{a-n}\right)\cos nx\right].$$

Agar $f(x)=\cos ax$ funksiya teoremaning shartlarini bajarishini e’tiborga olsak, unda

$$\cos ax=\frac{\sin a\pi}{\pi}\left[\frac{1}{a}+\sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{a+n}+\frac{1}{a-n}\right)\cos nx\right]$$

bo‘lishini topamiz.

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