Mathematics of economic processes nature and methods of modeling

Akmal I. Sotvoldiyev
akmal.sotvoldiyev@mail.ru
Tashkent Institute of Finance

Abstract: In the article, mathematical modeling is studied as one of the important methods in the study of the economy, and the issues of choosing one of the methods of economic statistics, econometrics, optimal solutions and adapting them to the studied problem are highlighted.

Keywords: economic processes, management programming, mathematical programming, economic mathematical models, macroeconomic models, dispersion analysis, correlation analysis, regression analysis, prediction evaluation

INTRODUCTION

In todays, mathematics and economics are considered as two separate fields of knowledge, each of which has its own objects and subjects of research. Nevertheless, mathematical models have their value in predicting the development of economic processes [1, 2, 3, 4, 5, 6]. Economic mathematical models, like other models, are implemented by constructing and studying another simpler analog of the object in order to study it. The practical issues of economic mathematical models can include forecasting the development of economic processes, analyzing economic objects, and developing management decisions in all fields. Therefore, mathematical modeling is one of the important methods in the study of modern economy.

Mathematical methods used in the study of economics can be divided into three groups:

1) methods of economic statistics (dispersion analysis, correlation analysis, regression analysis);

2) econometrics (macroeconomic models, production function, utility function, construction of inter-sectoral balance and statistics);

3) methods of obtaining optimal solutions (mathematical programming, control programming, logistic problems and problems).

MATERIALS AND METHODS

1) Dispersion analysis. Mainly, in the implementation of dispersion analysis

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{k} n_i \left(x_i - \overline{x}\right)^2$$

formula is used. Here $n=\sum_{i=1}^{k} n_i$ - the classifier of the studied object $X(x_1, x_2, ..., x_k)$ - the total size of the size; n_i - x_i element frequency; x_i - element of the quantity classifying the studied object i; $\overline{x}=\frac{1}{n}\sum_{i=1}^{k} n_i x_i$ - $X(x_1, x_2, ..., x_k)$ average value of magnitude. Dispersion, in many cases, represents the risk that may occur in the observed economic process, that is, it creates an opportunity to choose better objects in the studied economic processes.

2) Correlation and regression analysis. We consider the correlation analysis in the following matches. It is known that the fit in the form of Fig. 1 is called a function, and the $y=f(x)$ equality is appropriate here.

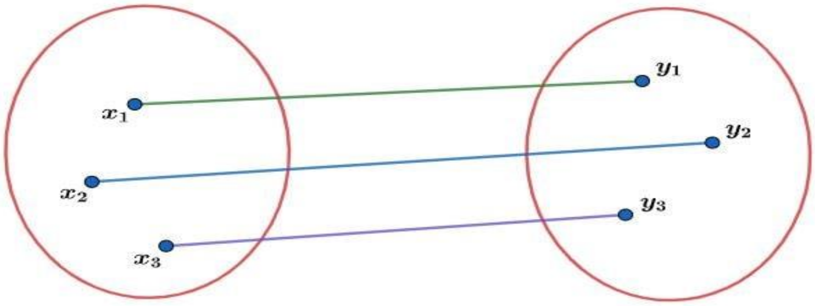

Figure 1. Functional connection

Therefore, the study of such processes, i.e. analysis, does not pose a great difficulty.

If the reflection is like Figure 2, then the $y=f(x)$ equality is not valid here f, which does not allow the possibility to find the appearance of the reflection, that is, regression analysis. Therefore, in such cases, it is necessary to perform a correlational analysis .

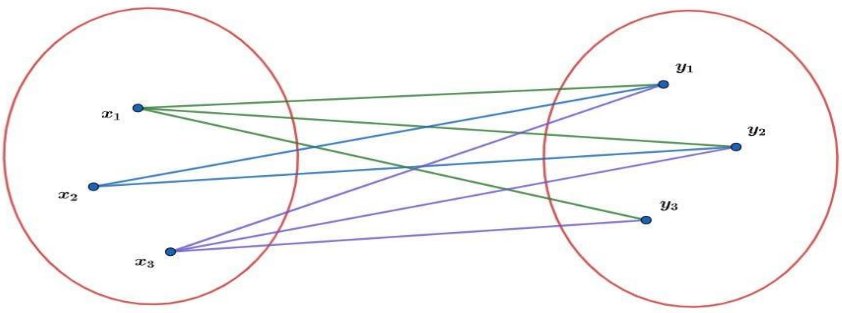

Figure 2. Statistical connection

We will consider the example in Figure 3, not in general, with the method of implementation of correlation analysis.

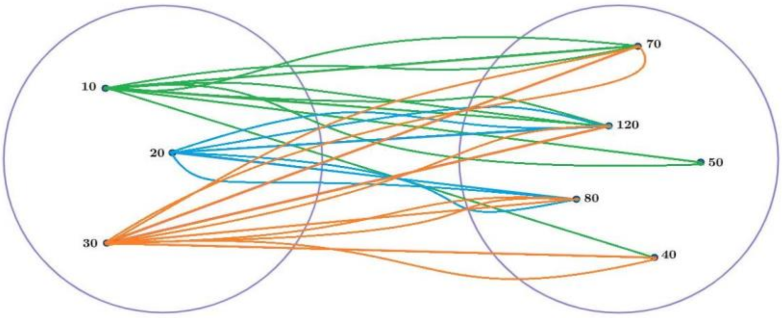

Figure 3. Statistical correlation in examples

For this, we will create a table (correlation table) that shows Figure 3:

Table 1

Y \ X	10	20	30	n_y
40	1	0	2	3
50	2	0	0	2
70	3	0	2	5
80	0	3	3	6
120	3	3	3	9
n_x	9	6	10	$n=25$

Using the table, y of x we can find the conditional mean of $\bar{y}_x$

1) $\bar{y}_{x=10} = \dfrac{40\cdot1+50\cdot2+70\cdot3+80\cdot0+120\cdot3}{9} = \dfrac{710}{9} \approx 79;$

2) $\bar{y}_{x=20} = \dfrac{40\cdot0+50\cdot0+70\cdot0+80\cdot3+120\cdot3}{6} = \dfrac{600}{6} = 100;$

3) $\bar{y}_{x=30} = \dfrac{40\cdot2+50\cdot0+70\cdot2+80\cdot3+120\cdot3}{10} = \dfrac{820}{10} = 82.$

As a result, the following fit is formed (Figure 4).

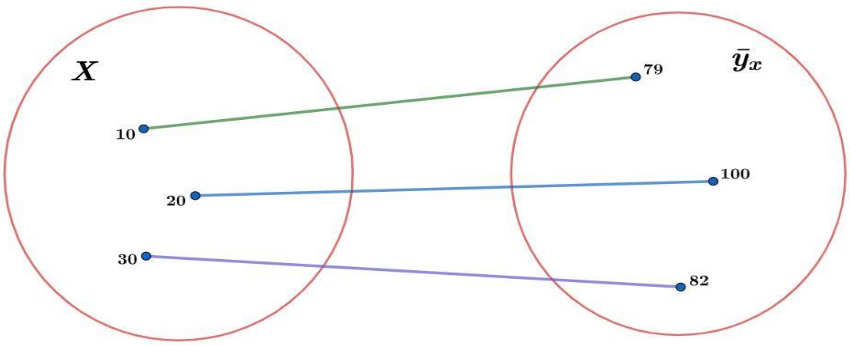

Figure 4. Correlation link

Here, we can now $\bar{y}_x = f(x)$ write the equation. Therefore, it f is possible to perform regression analysis, that is, the process of finding the appearance of reflection.

One of the main purposes of regression analysis is prediction. In many cases, we conclude our work by predicting. However, we ignore the fact that there is a continuation of our work, that is, to evaluate the level of accuracy of our model used in prediction.

RESULTS AND DISCUSSION

The estimation of the level of accuracy of the model is carried out using the relative error as follows:

y - observation value of the studied quantity (true value);

$\hat{y}$ - approximate value of the studied quantity (predicted value).

Using these values, absolute and relative errors are calculated:

$|y-\hat{y}|=\Delta y$ - absolute error;

$\delta=\frac{(y-\hat{y})^2}{y^2}\cdot 100\%$ - relative error.

Using these errors, we can estimate the accuracy of our model as follows:

$\delta\le 5\%$ - the level of accuracy of the model is good;

$\delta\le 15\%$ - the level of accuracy of the model is satisfactory;

$\delta>15\%$ - the level of accuracy of the model is unsatisfactory.

MACROECONOMIC MODELS

In most cases, ready-made macroeconometric models suitable for the studied economic process are selected. Therefore, when choosing a model, it is necessary to consider whether the process is continuous or discrete.

In continuous processes, models are expressed by differential equations. Common models for this condition include [6]:

1. Cobb-Douglas function (production function): $z=aK^mL^n,\ m+n=1$; here K - capital (Capital); L - labor (Labor).

To show that this function represents a continuous process (to show that it is continuous with time), we make some substitutions in it:

$$z=aK^mL^{-m}=aK^m\left(L_0e^{vt}\right)^{-m}=$$

$$=aL_0e^{vt}\left[\left(\frac{K_0^{1-m}}{L_0^{1-m}}-\frac{a(1-\rho)}{v+\mu}\right)e^{-(1-m)(v+\mu)t}+\frac{a(1-\rho)}{v+\mu}\right]^{\frac{m}{1-m}}.$$

2. Evans model. p - the price of the product is expressed through supply and demand as follows:

$$\frac{dp}{dt}=\gamma\left(D(t)-S(t)\right),\qquad \frac{dp}{dt}=-\gamma(b+d)p+\gamma(c-a).$$

Then the equilibrium price is determined by the following equation:

$$p^{*}=\lim_{t\to\infty}\left[\left(p_0-\frac{c-a}{b+d}\right)e^{-\gamma(b+d)t}+\frac{c-a}{b+d}\right]=\frac{c-a}{b+d}.$$

In discrete processes, models are represented by finite difference equations, and it is desirable to represent economic processes with discrete models. There are appropriate models for each process, for example, common models for forecasting include:

1. Brown model.

$$\hat{Y}_{t+1}=aY_t+(1-a)\hat{Y}_t$$

this model is used to predict step 2 based on step 1.

2. Holt model.

$$\begin{cases}\hat{Y}_{t+\tau}=a_t+\tau b_t\\ a_t=\alpha_1 Y_t+(1-\alpha_1)(a_{t-1}+b_{t-1})\\ b_t=\alpha_2(a_t-a_{t-1})+(1-\alpha_2)b_{t-1}\end{cases}$$

this model τ is used to predict the process after time, that is, after a long time:

Currently, the most widely used method for finding the optimal solution to economic problems is the Langrange method.

We assume that the utility function (for the consumer) is $U(x,y)=Cx^{\alpha}y^{\beta}$, $C\le 1,\quad \alpha+\beta\le 1$, and the condition limiting the consumer's choice (consumer's budget) should $p_1x+p_2y\le M$ be (p_1 - the price of the 1st product, p_2 - the price of the 2nd product, M - the consumer's budget).

There are 2 issues to consider here:

1) the issue of optimizing consumer selection;

2) the issue of determining consumer choice.

1) The problem of optimization of consumer selection is expressed as follows:

$$\begin{cases}\max\left\{Cx^{\alpha}y^{\beta}\right\}\\ p_1x+p_2y=M\end{cases}$$

$$x\ge 0,\quad y\ge 0,\quad \alpha+\beta\le 1.$$

It is appropriate to use the method of Lagrange multipliers to find a solution to such problems.

Using the Lagrangian function , we can make it look like this:

$$L(x,\ y,\ \lambda)=Cx^{\alpha}y^{\beta}+\lambda(M-p_1x-p_2y)\to\max$$

To find a solution to this problem, we create the following system of equations:

$$\begin{cases} L_x = U_x - \lambda p_1 = 0 \\ L_y = U_y - \lambda p_2 = 0 \\ L_\lambda = M - p_1 x - p_2 y = 0 \end{cases} \Rightarrow \begin{cases} U_x = \lambda p_1 \\ U_y = \lambda p_2 \\ M = p_1 x + p_2 y \end{cases} \Rightarrow \frac{U_x}{U_y} = \frac{p_1}{p_2}.$$

On the other hand, if we use the utility function,

$$U_x = \alpha C \frac{y^\beta}{x^{1-\alpha}}, \qquad U_y = \beta C \frac{x^\alpha}{y^{1-\beta}}.$$

In that case

$$\frac{U_x}{U_y} = \frac{\alpha y}{\beta x} = \frac{p_1}{p_2} \quad \Rightarrow \quad y = \frac{p_1 \beta}{p_2 \alpha} x.$$

Now $p_1 x + p_2 y = M$, using the boundary condition, we find the demand functions for products 1 and 2:

$$x^* = D_1(p_1, p_2, M) = \frac{M\alpha}{(\alpha+\beta)p_1}, \qquad y^* = D_2(p_1,\ p_2, M) = \frac{M\beta}{(\alpha+\beta)p_2}.$$

These functions show the effect of the consumer's budget and product price on his demand.

2) The issue of determining consumer choice is expressed as follows:

$$\begin{cases} \max\{p_1 x + p_2 y\} \\ C x^\alpha y^\beta = U \end{cases}$$

$$x \ge 0, \quad y \ge 0, \quad \alpha + \beta \le 1.$$

The method of Lagrange multipliers is used to solve this problem, and its solution $m(p_1, p_2, U)$ is expressed as a cost function.

CONCLUSION

Based on the above, we can make the following conclusion:

1. It is necessary to correctly understand the purpose of studying the economic process and the essence of the problem in it, and make sure that one of the above 3 groups corresponds to the problem we are studying and our purpose.

2. One of the models belonging to the selected group should be adapted to the problem under study.

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