To’rt o’lchamli qo’zg’alishga ega Fridrixs modeli uchun Fredgolm determinant

Sabina Ulug’bek qizi Saidova
Buxoro davlat universiteti

Annotatsiya: Ushbu maqolada to’rt o’lchamli qo’zg’alishga ega Fridrixs modeli uchun Fredgolm determinant qurilgan. Fridrixs modelining xos qiymatlari va Fredgolm determinantining nollari orasidagi bog’lanish isbotlangan.

Kalit so’zlar: Fridrixs modeli, diskret spektr, muhim spektr, Fredgolm determinant

Fredholm determinant for the Friedrichs model with four-dimensional excitation

Sabina Ulugbek qizi Saidova
Bukhara State University

Abstract: In this article, the Fredholm determinant for the Friedrichs model with four-dimensional excitation is constructed. The connection between the eigenvalues of the Friedrichs model and the zeros of the Fredholm determinant is proved.

Keywords: Friedrichs model, discrete spectrum, critical spectrum, Fredholm determinant

Matematik analiz, matematik fizika va ehtimollar nazariyasining qator masalalarida Fridrexs modeli deb nomlanuvchi operator paydo bo’ladi. Bu operator birinchi marta 1937 yilda K.Fridrixs tomonidan uzluksiz spektr qo’zg’alishlari nazariyasi modeli sifatida qaralgan [1].

$L_2[-1;1]$ Gilbert fazosida

$$H_\varepsilon f(x) = xf(x) + \varepsilon \int_{-1}^{1} v(x,y) f(y)\, dy$$

ko’rinishida ta’sir qiluvchi operatorni qaraymiz. Bu yerda ε haqiqiy musbat son, $v(x,y)$ esa $[-1;1]^2$ da aniqlangan haqiqiy qiymatli o’zining o’zgaruvchilari bo’yicha uzluksiz funksiya bo’lib, Gyolder sharti hamda quyidagi shartlarni qanoatlantiradi

$$v(-1,y) = v(1,y) = v(x,-1) = v(x,1) = 0.$$

Fridrixs tomonidan $\varepsilon \in R$ parametrning yetarlicha kichik qiymatlarida H_ε va H_0 operatorlar unitar ekvivalent ekanligi isbotlangan. 1948-yilda Fridrixs o'zining [2] ishida o'z modelini quyidagicha umumlashtirish masalasini taklif qilgan: birinchidan, $[-1; 1]$ o'rniga haqiqiy sonlar o'qidagi ixtiyoriy chekli yoki cheksiz bo'lgan l intervalni qarash; ikkinchidan, qiymatlari biror abstrakt Gilbert fazosi bo'lgan f funksiyalarni qarash. l interval cheksiz bo'lgan holda $v(x, y)$ yadro cheksizlikda kamayuvchi bo'lsin degan qo'shimcha shart kiritib Fridrixs bu nisbatan umumiy holda H_ε va H_0 operatorlarning unitar ekvivalent ekanligini isbotlagan. Keyinchalik Fridrixsning [1] va [2] ishlari O.A. Ladijenskiy, L.D.Faddeyevlar tomonidan [3] maqolada va L.D.Faddeyev tomonidan [4] maqolada rivojlantirilgan. Panjaradagi Fridrixs modeli bilan bog'liq tadqiqotlar [5-8] ishlarda olib borilgan. Umumlashgan Fridrixs modelining ayrim spektral xossalari [9-20] ishlarda o'rganilgan. Bu xossalar o'z navbatida panjaradagi soni saqlanmaydigan va uchtadan oshmaydigan zarrachalar sistemasiga mos 3-tartibli operatorli matrisalarning muhim va diskret spektrlarini tadqiq qilishda foydalanilgan.

$T^d - d$ o'lchamli tor va $L_2(T^d) - T^d$ da aniqlangan kvadrati bilan integrallanuvchi (kompleks qiymatli) funksiyalarning Hilbert fazosi bo'lsin. $L_2(T^d)$ Hilbert fazosida Fridrixs modeli deb nomlanuvchi A operatorni quyidagi tenglik yordamida aniqlaymiz

$$A \coloneqq A_0 - A_1 - A_2 - A_3 - A_4 \quad (1)$$

bu yerda A_0 va $A_i, i = \overline{1,4}$ operatorlar

$$(A_0 f)(p) = u(p) f(p), (A_{\,i} f)(p) = \mu_i \, \delta_i(p) \int_{T^d} \delta_i(t) f(t) dt, i = 1$$

tengliklar bilan aniqlangan. Bunda $\mu_i > 0, i = \overline{1,4} -$ ta'sirlashish parametri; $u(\cdot)$ va $\delta_i(\cdot)$, $i = \overline{1,4} - T^d$ da aniqlangan uzluksiz funksiyalar.

Yuqoridagi shrtlarda (1) tenlik bilan aniqlangan A operator chegaralangan va o'z-o'ziga qo'shma operator bo'ladi.

Aniqlanishiga ko'ra A_0 operatorning qo'zgalish operatori

$$A_1 + A_2 + A_3 + A_4$$

rangi 4 ga teng bo'lgan o'z-o'ziga qo'shma operator bo'ladi. Chekli o'lchamli qo'zg'alishlarda muhim spektrning saqlanishi haqidagi Veyl teoremasiga ko'ra

$$\sigma_{ess}(A) = \sigma_{ess}(A_0)$$

tenglik o'rinli. A_0 operator $u(\cdot)$ funksiyaga ko'paytirish operatori bo'lib, uning muhim spektri uchun

$$\sigma_{ess}(A_0) = [m, M]$$

kabi aniqlanadi. Bu yerda

$$m \coloneqq \min_{p \epsilon T^d} u(p) \, , M \coloneqq \max_{p \epsilon T^d} u(p).$$

Oxirgi mulohazalardan,

$$\sigma_{ess}(A) = [m, M]$$

tenglik o'rinli bo'lishi kelib chiqadi.

$\mathbb{C}\backslash[m, M]$ to'plamda regulyar bo'lgan

$$I_{ij}(z) := \int_{T^3} \frac{v_i(t)v_j(t)dt}{u(t) - z}, i, j = \overline{1,4};$$

$$\Delta(z) := \det\left(\delta_{\ ij} - \mu_j I_{ij}(z)\right)^{\ 4}_{\ i,j=1},$$

funksiyalarni qaraymiz. Bu yerda $\delta_{\ ij}$ Kroniker simvoli, ya'ni

$$\delta_{\ ij} := \begin{cases} 1, agar\ i = j \\ 0, agar\ i \neq j \end{cases}.$$

Odatda, $\Delta(z)$ funksiyaga A operatorga mos Fredgolm determinant deb ataladi.

A operatorning xos qiymati va $\Delta(\cdot)$ funksiyaning nollari orasidagi bog'lanishni ifodalovchi tasdiqni keltiramiz.

1-lemma. $z \in \mathbb{C}\backslash[m, M]$ soni A operatorning xos qiymati bo'lishi uchun $\Delta(z) = 0$ tenglik o'rinli bo'lishi zarur va yetarlidir .

Isbot. $z \in \mathbb{C}\backslash[m, M]$ soni A operatorning xos qiymati, $f \epsilon L_2(T^d)$ unga mos xos funksiya bo'lsin. U holda f funksiya

$$u(p)f(p) - \sum_{i=1}^{4} \mu_i v_i(p) \int_{T^d} v_i(t)f(t)dt = zf(p) \ (2)$$

tenglikni qanoatlantiradi.

Agar

$$C_i := \int_{T^d} v_i(t)f(t)dt, i = \overline{1,4} \ (3)$$

kabi belgilash kiritsak (2) tenglik

$$u(p)f(p) - \sum_{i=1}^{A} \mu_i \vartheta_{\ i} c_i = z\, f(p)$$

ko'rinishiga keladi.

$z \in \mathbb{C}\backslash[m, M]$ ekanligini inobatga olsak, u holda ixtiyoriy $p \in T^d$ uchun $u(p) - z \neq 0$ munosabat o'rinli bo'lib, (4) tenglikdan $f(\cdot)$ uchun

$$f(p) = \frac{1}{u(p) - z} \sum_{i=1}^{4} \mu_i C_i v_i(p), (5)$$

tenglikni hosil qilamiz.

$f(\cdot)$ uchun topilgan (5) ifodani (3) tenglikga qo'yib, quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} \left(1-\mu_1 I_{11}(z)\right)C_1-\mu_2 I_{12}(z)C_2-\mu_3 I_{13}(z)C_3-\mu_4 I_{14}(z)C_4=0 \\ -\mu_1 I_{21}(z)C_1+\left(1-\mu_2 I_{22}(z)\right)C_2-\mu_3 I_{23}(z)C_3-\mu_4 I_{24}(z)C_4=0 \\ -\mu_1 I_{31}(z)C_1-\mu_2 I_{32}(z)C_2+\left(1-\mu_3 I_{33}(z)\right)C_3-\mu_4 I_{34}(z)C_4=0 \\ -\mu_1 I_{31}(z)C_1-\mu_2 I_{32}(z)C_2-\mu_3 I_{43}(z)C_3+\left(1-\mu_4 I_{44}(z)\right)C_4=0 \end{cases}$$

Bu tenglamalar sitemasini quyidagi matritsaviy tenglama bilan almashtiramiz:

$$\left(\delta_{\,ij}-\mu_j I_{ij}(z)\right)_{i,j=1}^{4}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{pmatrix}=0$$

Bu tenglama nolmas $(C_1, C_2, C_3, C_4)\in\mathbb{C}^4$ yechimga ega bo'lishi uchun $\Delta(z)=0$ bo'lishi zarur va yetarlidir. Agar $C_1=C_2=C_3=C_4=0$ bo'lsa, u holda $f(p)=0$ bo'ladi bu esa z ning A operator uchun xos qiymat ekanligiga zid. 1-lemma to'liq isbotlandi.

Bu tasdiqdan A operatorning diskret spektri uchun quyidagicha munosabat hosil bo'ladi:

$$\sigma_{disc}(A):=\{z\in\mathbb{C}\backslash[m,M]\mid\Delta(z)=0\}.$$

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