Fermionli Fok fazosidagi matritsaviy model operatorga mos Fredgolm determinanti

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Annotatsiya: Ushbu maqolada dastlab Fok fazosiga ta'rif berilib, uning qirqilgan qism fazosi haqida ma'lumotlar aytib o'tilgan. Fok fazosidan olingan elementlarning skalyar ko'paytmasi va normasi ko'rsatilib o'tilgan. Fok fazosining qirqilgan qism fazosida aniqlangan operatorli matritsa uchun Fredgolm determinant qurilgan.

Kalit so'zlar: kompleks sonlar fazosi, Hilbert fazosi, Fok fazosi, qirqilgan qism fazo, skalyar ko'paytma, norma, operatorli matritsa

Fredholm determinant corresponding to a matrix model operator in Fock space with fermions

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Abstract: This paper first defines the Fock space and gives information about its truncled subspaces. The scalar product and norm of the elements taken from the Fock space are shown. The Fredholm determinant is constructed for the matrix where the operator is defined in the cut subspace of the Fock space.

Keywords: space of complex numbers, Hilbert space, Fock space, cut subspace, scalar product, norm, operator matrix

$\mathbb{C}$– bir o'lchamli kompleks sonlar fazosi bo'lsin. Ixtiyoriy $n \in N$ natural soni uchun $L_2[a,b]^n$ orqali $[a,b]^n$ da aniqlangan kvadrati bilan integrallanuvchi (umuman olganda kompleks qiymatli) funksiyalarning Hilbert fazosini belgilaymiz.[1-4] Quyidagicha belgilashlar kiritamiz:

$$H_0 := C, H_n := L_2([a,b]^n), n \in N;$$
$$H^{(m)} := \bigoplus_{n=0}^{m-1} H_n\,, H := \bigoplus_{n=0}^{\infty} H_n.$$

Ta'rif. H Hilbert fazoga Fok fazosi deyiladi, $H^{(m)}$ Hilbert fazosiga esa Fok fazosining qirqilgan m – zarrachali qism fazosi deyiladi.

Shunday qilib,

$$H^{(1)} := H_0 \oplus H_1 = \{(f_0, f_1): f_k \in H_k, k = 0,1\};$$

$$H^{(2)} := H_0 \oplus H_1 \oplus H_2 = \{(f_0, f_1, f_2): f_k \in H_k, k = 0,1,2\};$$

$$H^{(3)} := H_0 \oplus H_1 \oplus H_2 \oplus H_3 = \{(f_0, f_1, f_2, f_3): f_k \in H_k, k = 0,1,2,3\};$$

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$$H^{(m)} := H_0 \oplus H_1 \oplus \ldots \oplus H_{m-1} = \{(f_0, f_1, \ldots, f_m): f_k \in H_k, k = \overline{0,m}\}.$$

Odatda, $L_2[a,b]$ fazo yordamida qurilgan Fok fazosi $F(L_2[a,b])$ kabi belgilanadi.

$m \in N$ - fiksirlangan natural son bo‘lsin. Ixtiyoriy ikkita

$f = (f_0, f_1, \ldots, f_m) \in H^{(m)}$ va $g = (g_0, g_1, \ldots, g_m) \in H^{(m)}$ vektor – funksiyalar uchun ularning skalyar ko‘paytmasi

$$(f, g) = (f_0, g_0)_0 + (f_1, g_1)_1 + \ldots + (f_m, g_m)_m$$

kabi aniqlanadi, bu yerda

$$(f_0, g_0)_0 = f_0 \cdot \overline{g_0};\ (f_k, g_k)_k = \int_a^b f_k(t) \cdot \overline{g_k(t)} dt, k = \overline{1,m}.$$

Xuddi shuningdek, $f = (f_0, f_1, \ldots, f_m) \in H^{(m)}$ vektor – funksiyaning normasi

$$\|f\| = \sqrt{\sum_{k=0}^{m} \|f_k\|_k^2}$$

tenglik yordamida aniqlanadi [5-8], bunda

$$\|f_0\|_0 := |f_0|$$

$$\|f_k\|_k := \sqrt{\int_a^b |f_k(t)|^2 dt}, k = \overline{1,m}.$$

Endi H Fok fazosida skalyar ko‘paytma va normani aniqlaymiz. Ixtiyoriy ikkita

$$F = (f_0, f_1, \ldots, f_n, \ldots) \in H \text{ va } G = (g_0, g_1, \ldots, g_n, \ldots) \in H$$

elementlar uchun ularning skalyar ko‘paytmasi

$$(F, G) := \sum_{k=0}^{\infty} (f_k, g_k)_k$$

kabi, F vektor – funksiya normasi esa

$$\|F\| := \sqrt{\sum_{k=0}^{\infty} \|f_k\|_k^2}$$

kabi aniqlanadi. [13-14]

$\mathcal{H}$ Hilbert fazosida ta’sir qiluvchi quyidagi ya’ni $H^{(3)}$ Hilbert fazosida quyidagi

$$A = \begin{pmatrix} A_{00} & A_{01} & 0 \\ A_{01}^* & A_{11} & A_{12} \\ 0 & A_{12}^* & A_{22} \end{pmatrix} \quad (1)$$

3×3 operatorli matritsani qaraymiz, bu yerda $A_{ij}: H_j \to H_i$, $i, j = 0,1,2$.

Operatorlar ushbu tengliklar orqali ta'sir qiladi:

$$(A_{00}f_0)_0 = \omega_0 f_0;$$
$$(A_{01}f_1)_0 = \int_{L_2[a;b]} v(t) f_1(t)\, dt;$$
$$(A_{11}f_1)_1(p) = \omega_1(p) f_1(p);$$
$$(A_{12}f_2)_1(p) = \int_{L_2[a;b]} v(t) f_2(p,t) dt;$$
$$(A_{22}f_2)_2(p,q) = \omega_2(p,q) f_2(p,q)$$

$f_i \in H_i$, $i = 0,1,2.$, ω_0 fiksirlangan haqiqiy son, $\omega_1(\cdot)$ va $v(\cdot)$ funksiyalar $L_2[a,b]$ da aniqlangan haqiqiy qiymatli uzluksiz funksiyalar, $\omega_2(\cdot,\cdot)$esa $L_2[a,b]^2$ da aniqlangan haqiqiy qiymatli simmetrik uzluksiz funksiya.

$m = 3$ bo'lgan holda A_{01}^* va A_{12}^* operatorlarning aniq ko'rinishlarini quyidagicha topiladi

$$(A_{01}^* f_0) = v(t) f_0;$$

$v(t)$ - haqiqiy o'zgaruvchili funksiya. Haqiqiy sonning qo'shmasi o'ziga teng ekanligidan oxirgi tenglikni quyidagicha yozish mumkin.

$$(A_{12}^* f_1)(x,y) = \frac{v(y) f_1(x) + v(x) f_1(y)}{2}$$

Endi A operatorning muhim va diskret spektrlarini o'rganish maqsadida umumlashgan Fridrixs modeli deb ataluvchi operatorni kiritamiz. Bu operator $H_0 \oplus H_1$ Gilbert fazosida

$$A_1 := \begin{pmatrix} A_{00} & A_{01} \\ A_{01}^* & A_{11} \end{pmatrix}$$

kabi aniqlangan. Uning $A_{ij}, i \le j, i, j = 0,1$ matritsaviy elementlari yuqorida keltirilgan. A_1 operator chiziqli, chegaralangan va o'z-o'ziga qo'shma bo'ladi.

$H_0 \oplus H_1$ Gilbert fazosida A_1^0 operatorni qaraymiz va bu operator quyidagicha aniqlangan bo'lsin:

$$A_1^0 := \begin{pmatrix} 0 & 0 \\ 0 & A_{11} \end{pmatrix}$$

Aniqlanishiga ko'ra, A_1^0 operatorli matritsaning muhim spektri A_{11} operatorli matritsaning muhim spektri bilan ustma-ust tushadi hamda A_{11} operator ko'paytirish operatori ekanligidan foydalansak, A_{11} operatorning spektri ko'paytirilayotgan funksiyaning qiymatlar sohasiga teng ekanligini aniqlanishimiz mumkin ya'ni,

$$\sigma_{ess}(A_1^0) = [m; M], \sigma_{disk}(A_1^0) = \{0\}.$$

bu yerda

$$m = \min_{p \in L_2[a;b]} \omega_1(p)\ , M = \max_{p \in L_2[a;b]} \omega_1(p)$$

munosabat o'rinli.

Yuqorida bayon qilingan mulohazalarni hisobga olsak, A_1 operatorli matritsaning muhim spektri uchun quyidagi tenglik o'rinli ekan:

$$\sigma_{ess}(A_1) = \sigma_{ess}(A_1^0) = [m; M]$$

$C\backslash[m; M]$ sohada regulyar bo'lgan

$$\Delta(z) = \omega_0 - z - \int_{L_2[a;b]} \frac{v^2(t)}{\omega_1(t) - z} dt$$

funksiyani kiritamiz. Odatda $\Delta(\cdot)$funksiyaga A_1 umumlashgan Fridrixs modeliga mos keluvchi Fredgolm determinant deyiladi.

Lemma. $z \in C\backslash[m; M]$ soni A_1 operatorli matritsaning xos qiymati bo'lishi uchun $\Delta(\cdot) = 0$ bo'lishi zarur va yetarlidir.

Isbot: Lemmani isbotlash maqsadida A_1 operatorli matritsaning $f = (f_0, f_1)$ vektorga ta'sirining ko'rinishi aniqlaymiz:

$$A_1 f := \begin{pmatrix} A_{00} f_0 + A_{01} f_1 \\ A_{01}^* f_0 + A_{11} f_1 \end{pmatrix}$$

$A_1 f$ opeartorli matritsaning elementlari yuqorida berilgan shartlarni qanoatlantiradi va $A_1 f = zf, z \in C\backslash[m; M]$ xos qiymatga mos tenglamani qaraymiz.

$$\begin{cases} A_{00} f_0 + A_{01} f_1 = z f_0 \\ A_{01}^* f_0 + A_{11} f_1 = z f_1 \end{cases}$$

ya'ni,

$$\begin{cases} \omega_0 f_0 + \int_{L_2[a;b]} v(t) f_1(t)\, dt = z f_0 \\ v(p) f_0 + \omega_1(p) f_1(p) = z f_1(p) \end{cases} \quad (2)$$

(2) tenglamalar sistamasining ikkinchi tenglamasidan f_1 vektor funksiyani topib olamiz:

$$f_2(p, q) = \frac{v(p) f_0}{z - \omega_1(p)} \quad (3)$$

(3) tenglamani 1- tenglamalar sistemasining birinchi tenglamasiga qo'yib, f_0 vektor funksiyani topib olamiz:

$$\omega_0 f_0 + \int_{L_2[a;b]} v(t) \frac{v(t) f_0}{z - \omega_1(t)} dt = z f_0\, ;$$

$$\left(\omega_0 - z - \int_{L_2[a;b]} \frac{v^2(t)}{\omega_1(t) - z} dt \right) f_0 = 0$$

Oxirgi tenglik o'rinli bo'lishi uchun ko'paytuvchilardan kamida bittasi nolga teng bo'lishi kerak. Ammo vektor funksiyaning ta'rifiga ko'ra f_0 vektor funksiya noldan farqli demak,

$$\Delta(z) = \omega_0 - z - \int_{L_2[a;b]} \frac{v^2(t)}{\omega_1(t) - z} dt$$

ko'rinishida aniqlangan funksiya nol bo'lishi kerak.

Lemma isbotlandi.

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