Ikki o'lchamli statsionar konvektsiya-diffuziya tenglamasi uchun Dirixle masalasini sonli yechish

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Annotatsiya: Ushbu maqolada ikki o'lchamli konvektsiya-diffuziya masalasi sonli yechilgan. Masalani yechish uchun bir nechta ayirmali sxemalar qo'llangan. Masalani sonli yechish algoritmi asosida Python tilida dastur tuzilgan va sonli natijalar olingan. Sonli natijalar asosida grafiklar chizilgan.

Kalit so'zlar: ikki o'lchamli, konveksiya-diffuziya, statsionar, tenglama

Numerical solution of the Dirichlet problem for the two-dimensional stationary convection-diffusion equation

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Abstract: In this article, the two-dimensional convection-diffusion problem is solved numerically. Several different schemes have been used to solve the problem. Based on the algorithm of numerical solution of the problem, a program was created in Python and numerical results were obtained. Graphs are drawn based on numerical results.

Keywords: two-dimensional, convection-diffusion, stationary, equation

1. Kirish

Konvektsiya-diffuziya masalalari suyuqlik va gaz mexanikasi matematik modellari uchun tipik masalalar hisoblanadi. Bu masalalarda issiqlik, aralashma tarqalishi faqatgina diffuziya hisobiga emas, balki harakatdagi muhitga ham bog'liq. Hisoblash gidro va gazodinamikasi, issiqlikmassaalmashinuvi jarayonlarini sonli modellashtirish [1, 2, 3, 4, 5] larda bayon etilgan. Diffuziya masalalarini sonli modellashtirish ancha yaxshi o'rganilgan [6, 7, 8, 9, 10]. Ammo konvektsiya-diffuziya masalasini sonli yechish unch yaxshi o'rganilmagan. Konvektsiya-diffuziya masalasini sonli yechish bo'yicha [11, 12, 13] adabiyotlarga murojat qilish mumkin.

Ushbu ishda to'g'ri to'rtburchak sohada statsionar konvektsiya-diffuziya tenglamasi uchun Dirixle masalasini sonli yechish masalasini qaraymiz. Masalani yechish uchun yuqori relaksatsiyali iteratsiya usuli qo'llanilgan. Masalani yechish uchun Python algoritmik tilida dastur tuzilgan va hisoblash natijalari grafik ko'rinishda keltirilgan.

2. Masalaning qo'yilishi va ayirmali sxema

Ushbu to'g'ri to'rtburchakda

$$\Omega = \{(x, y), 0 \leq x \leq l_1, 0 \leq y \leq l_2,\}$$

ikkinchi tartibli statsionar konvektsiya-diffuziya tenglamasi uchun Dirixle masalasi yechiladi:

$$-\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + b(x, y)\frac{\partial u}{\partial x} = f(x, y), x, y \in \Omega, \quad (1)$$

$$u(x, y) = \mu(x, y), x, y \in \partial\Omega. \quad (2)$$

(1), (2) Dirixle masalasini sonli yechish uchun yuqori relaksatsiyali iteratsion usulni qo'llaymiz. (1), (2) masalani yechayotganda konvektiv hadni approksimatsiya qilishda markaziy ayirmali hosiladan foydalanamiz. Masalani $f(x, y) = 1$, $\mu(x, y)$ =0, $b(x, y)$ =0; 2,5; 5,0 va 10,0 qiymatlarda sonli modellashtiramiz. Ω sohada to'rni quyidagicha kiritamiz:

$$\omega_{h_1 h_2} = \left\{(x_i, y_j),\ x_i = ih_1, i = \overline{0, N_1}, h_1 = \frac{l_1}{N_1},\right.$$
$$\left. y_j = jh_2, j = \overline{0, N_2}, h_2 = \frac{l_2}{N_2}\right\}.$$

(1) tenglamani $\omega_{h_1 h_2}$ to'rda approksimatsiyalab quyidagi ayirmali tenglamaga ega bo'lamiz:

$$\frac{y_{i-1,j} - 2y_{i,j} + y_{i+1,j}}{h_1^2} + \frac{y_{i,j-1} - 2y_{i,j} + y_{i,j+1}}{h_2^2} -$$
$$-b_{i,j}\frac{y_{i+1,j} - y_{i-1,j}}{2h_1} = -f_{i,j}, i = \overline{1, N_1 - 1}, j = \overline{1, N_2 - 1} \quad (3)$$

(3) tenglamani quyidagi ketma-ket yuqori relaksatsiyali iteratsion usulini qo'llaymiz. Bu usul yordamida iteratsion jarayon quyidagi ko'rinishda yoziladi:

$$y_{i,j}^{(s+1)} = y_{i,j}^{(s)} + \frac{\omega}{\frac{2}{h_1^2} + \frac{2}{h_2^2}}\left(\frac{y_{i-1,j}^{(s)} - 2y_{i,j}^{(s)} + y_{i+1,j}^{(s)}}{h_1^2} +\right.$$
$$\left. + \frac{y_{i,j-1}^{(s)} - 2y_{i,j}^{(s)} + y_{i,j+1}^{(s)}}{h_2^2} - b_{i,j}^{(s)}\frac{y_{i+1,j}^{(s)} - y_{i-1,j}^{(s)}}{2h_1} + f_{i,j}^{(s)}\right) = \quad (4)$$
$$= y_{i,j}^{(s)} + \frac{\omega}{d} r_{i,j}^{(s)},$$

bu yerda ω parameter $1 \leq \omega \leq 2$ sohada topiladi. Yuqori relaksatsiyali iteratsion usulda butun to'rda $\left|r_{i,j}^{(s)}\right| < \varepsilon$ tengsizlik bajarilguncha iteratsiya davom etadi.

$b(x, y)$ =0 bo‘lganda Puasson tenglamasi uchun Dirixle masalasida yuqori relaksatsiya usulining ω ning optimal parametri quyidagi formula bilan topiladi:

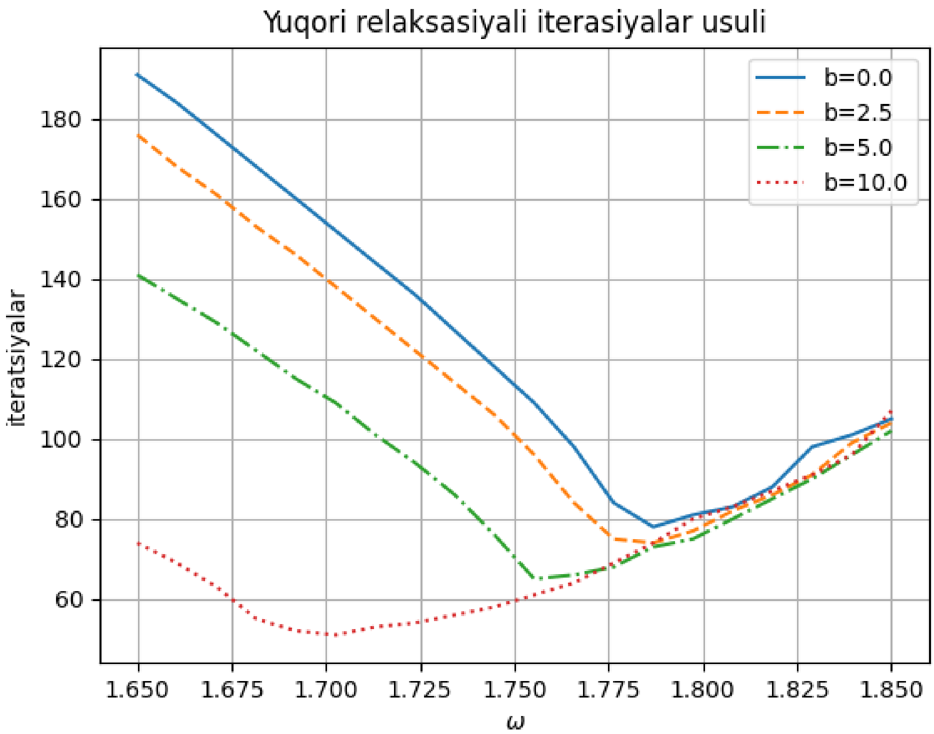
$$\omega = \frac{4}{2 + \sqrt{4 - \left(\cos\frac{\pi}{N_1 - 1} + \cos\frac{\pi}{N_2 - 1}\right)^2}}. \quad (5)$$

3. Sonli natijalar, ularning tahlili va xulosalar

Ammo statsionar konveksiya-diffuziya tenglamasi uchun ω ning optimal parametrini topishning umumiy formulasi mavjud emas, shu sababli iteratsion jarayon uchun tuzilgan dasturni ishlatib $b(x, y)$ =2,5; 5,0 va 10,0 qiymatlarida ω ning mos optimal (minimal iteratsiyaga ega bo‘lgan) qiymatlari topiladi.

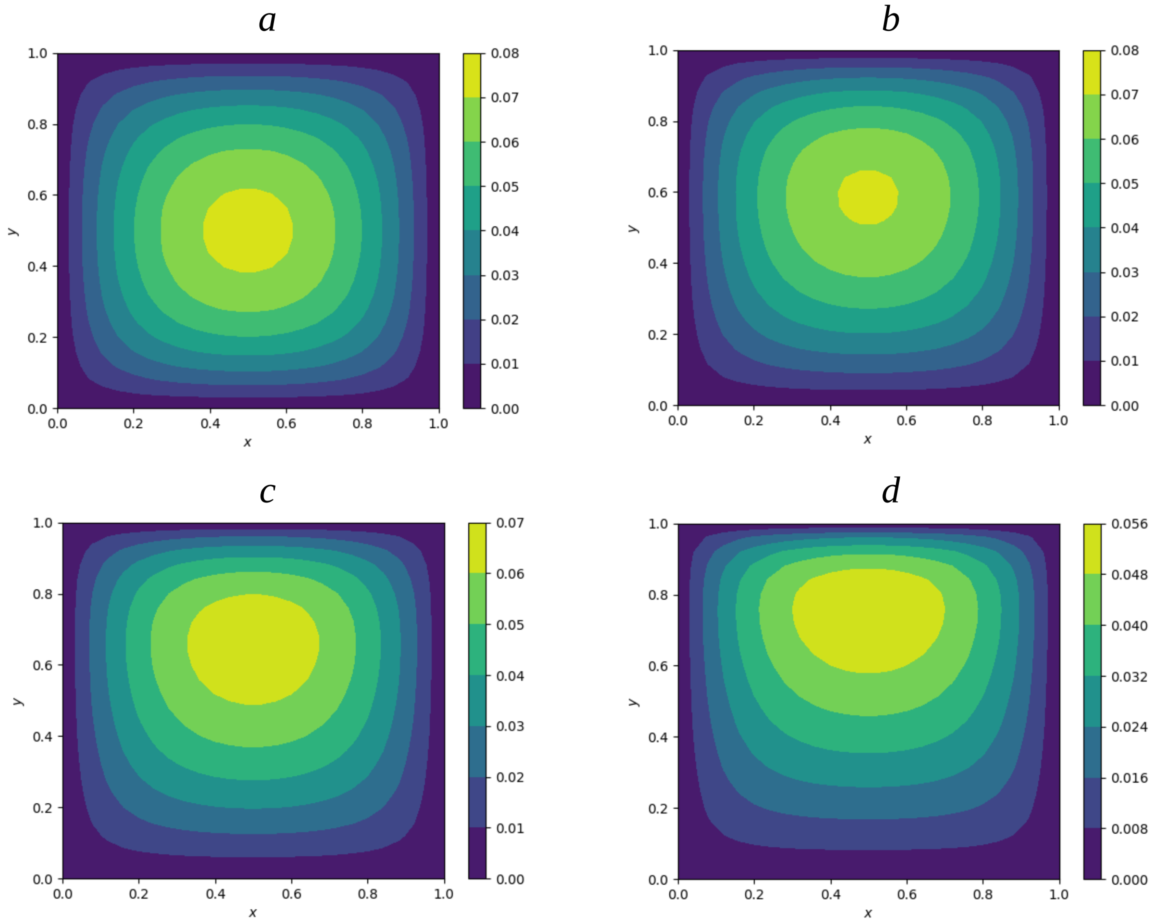
(1), (2) masalani birlik kvadratda $l_1 = l_2 = 1$ va $h_1 = h_2 = 0{,}04$ (4) iteratsion jarayon asosida Python algoritmik tilini qo‘llab sonli yechish dasturi tuzildi.

Statsionar konvektsiya-diffuziya tenglamasi uchun Dirixle masalasini yuqori relaksatsiyali usul bo‘yicha $b(x, y)$ ning turli qiymatlarida ω parametrning optimal qiymatlarini topish sonli natijalar 1-rasmda keltirilgan. Sonli natijalarga ko‘ra $b(x, y)$ =0,0; 2,5; 5,0 va 10,0 qiymatlarga mos ω parametrning optimal qiymatlari ω =1,78; 1,773; 1,753 va 1,70 larga teng bo‘ldi. 1-rasmda $b(x, y)$ nig turli qiymatlaridagi ω parametrning optimal qiymatlari yaqqol ko‘rinib turibdi.



1-rasm. Yuqori relaksatsiya usulining ω ning turli qiymatlaridagi iteratsiyalar soni

2-rasmda ω parametrning optimal qiymatlarida statsionar konvektsiya-diffuziya masalasi sonli yechimi sath chiziqlari ko‘rinishida tasvirlangan. 2-rasmdagi sonli natijalarda konveksiyaning ta’siri yaqqol ko‘rinib turibdi. Sonli natijalarning ko‘rsatishicha konvektiv had oldidagi koeffitsiyentning oshishi bilan maksimal iteratsiyalar soni kamayishi ko'rinib turibdi. $b(x,y)$ =0,0; 2,5; 5,0; 10,0 qiymatlarga mos ω =1,78; 1,773; 1,753; 1,70 optimal qiymatlarida maksimal iteratsiyalar soni mos ravishda s =74, 70, 61, 51 ga teng bo'ldi (mos ravishda 2-rasm, *a, b, c, d*).



2-rasm. Statsionar konvektsiya-diffuziya tenglamasi sonli yechimi: $b=0{,}0$, $s=74$, $\omega=1{,}78$ (a); $b=2{,}5$, $s=70$, $\omega=1{,}773$ (b); $b=5{,}0$, $s=61$, $\omega=1{,}753$ (c); $b=10{,}0$, $s=51$, $\omega=1{,}70$ (d)

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