Bir Fridrixs model operatorining muhim spektrdan tashqaridagi xos qiymatlari soni va joylashgan o’rni

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Annotatsiya: Ushbu maqolada zarrachalar ta’sir energiyalari $\gamma,\mu\epsilon\mathbb{R}$ va kvaziimpuls k$\epsilon\mathbb{T}^d$ ning $H_{\gamma\mu}$(k), k $\epsilon\mathbb{T}^d$ operator yagona yoki ikkita xos qiymatga bo‘ladigan qiymatlari ajratib ko‘rsatilgan, $H_{\gamma\mu}$(k), k $\epsilon\mathbb{T}^d$ d=1,2 operatorning ikkinchi xos qiymati mavjudligi paydo qiluvchi va yo‘qotuvchi operatorlarga bog‘liqligi o‘rganilgan, xos qiymatlar $E^{(i)}{}_{\gamma\mu}$(k), i=1,2 larning joylashish o’rni parametrlarga bog‘liq o‘rganilgan.

Kalit so‘zlar: kompleks fazo, vektor fazo, qism fazo,qarama-qarshi element, normallangan fazo, Banax fazosi, norma, ortogonal element, separabel hilbert fazosi, chegaralangan operator, chiziqli operator, Lebeg integrali, teskari operator, o‘z-o‘ziga qo‘shma operator, kompakt operator, nisbiy kompakt, xos qiymat, spektr, unitar operator,muhim spektr, diskret spektr, qoldiq spektr, absolyut yaqinlashuvchi qator, Gamiltonian, ikki zarrachali gamiltonian, birlik operator, integral operator, Veyl teoreamasi, Shryodinger operatori, Fridrixs operatori

Number and location of eigenvalues of a Friedrichs model operator outside the critical spektrum

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Abstract: In this article the values of $H_{\gamma\mu}$(k), k $\epsilon\mathbb{T}^d$ of particle impact energies $\gamma,\mu\epsilon\mathbb{R}$ and quasi-impulse k$\epsilon\mathbb{T}^d$, which the operator divides into single or two eigenvalues, $H_{\gamma\mu}$(k), k $\epsilon\mathbb{T}^d$ d=1,2 the presence of the second eigenvalue of the operator depends on the appearing and disappearing operators, The position of eigenvalues $E^{(i)}{}_{\gamma\mu}$(k), i=1,2 was studied depending on the parameters.

Keywords: complex space, vector space, partial space, opposite element, normalized space, Banat space, norm, orthogonal element, separable Hilbert space, bounded operator, linear operator, Lévesque integral, inverse operator, self-adjoin common operator, compact operator, relatively compact, eigenvalue, spectrum,

unitary operator, critical spectrum, discrete spectrum, residual spectrum, absolute convergent series, Hamiltonian, two-particle Hamiltonian, unitary operator, integral operator, Weill theorem, Schrodinger operator, Fredrik's operator

1. $H_{\gamma\mu}(k)$ Fredrick's model operator.

Faraz qilamiz, $\mathrm{H}=\mathrm{H}_0\oplus\mathrm{H}_1$ – bir o'lchamli $\mathrm{H}_0=C^1$ kompleks sonlar Gilbert fazosi (1-kanal) va $\mathrm{H}_1=L_e^2(\mathrm{T}^d)$ – $d\geq 1$ o'lchamli tor T^d da kvadrati bilan integrallanuvchi juft funksiyalarning Gilbert fazosi (2-kanal) ning to'g'ri yig'indisidan iborat ikki kanalli Gilbert fazosi bo'lsin.

$E(k),\ \ k\in\mathrm{T}^d$ orqali H_0 Gilbert fazosidagi $\varepsilon(k)=4\sum_{i=1}^{d}(1+\cos\frac{k^{(i)}}{2})$ songa ko'paytirish operatorini belgilaymiz [1] :

$$E(k)\,f_0=\varepsilon(k)\,f_0,\quad f_0\in\mathrm{H}_0,$$

$h_\mu(k),\ \ k\in\mathrm{T}^d$ – H_1 Gilbert fazosidagi kontakt ta'sirlashuvchi ikkita bir xil zarracha (bozon) li sistemaga mos ikki zarrachali diskret Shroedinger operatori bo'lsin:

$$h_\mu(k)=h_0(k)+\mu V,\quad \mu\geq 0,$$

bunda $h_0(k)$ operator $\varepsilon_k(\cdot)$ funksiyaga ko'paytirish operatori:

$$(h_0(k)\,f_1)(q)=\varepsilon_k(q)\,f_1(q),\quad f_1\in\mathrm{H}_1, \tag{1}$$

$$\varepsilon_k(q)=\varepsilon\left(\frac{k}{2}-q\right)+\varepsilon\left(\frac{k}{2}+q\right)=2\sum_{i=1}^{d}\left(1-\cos\frac{k^{(i)}}{2}\cos q^{(i)}\right)$$

hamda V bir o'lchamli integral operator:

$$(Vf_1)(q)=\alpha_0(f_1,\alpha_0)_{\mathrm{H}_1}=\frac{1}{(2\pi)^d}\int_{\mathrm{T}^d}f_1(s)ds,\quad f_1\in\mathrm{H}_1,\quad \alpha_0\equiv(2\pi)^{-d/2}.$$

$H_{\gamma\mu}(k),\ k\in\mathrm{T}^d,\ \gamma\in\mathrm{R},\ \mu\in\mathrm{R}$ operator H gilbert fazosida

$$H_{\gamma\mu}(k)=\begin{pmatrix}E(k) & C_\gamma^*\\ C_\gamma & h_\mu(k)\end{pmatrix} \tag{2}$$

formula yordamida aniqlanadi, bunda

$$C_\gamma:\mathrm{H}_0\to\mathrm{H}_1,\quad C_\gamma f_0=\gamma\alpha_0(f_0,\alpha_0)_{\mathrm{H}_0}$$

va

$$C_\gamma^*:\mathrm{H}_1\to\mathrm{H}_0,\quad C_\gamma^* f_1=\gamma(f_1,\alpha_0)_{\mathrm{H}_1}$$

operatorlar mos ravishda paydo qiluvchi va yo'qotuvchi operatorlar.

Tushunarliki, $H_{\gamma\mu}(k),\ \gamma \in \mathrm{R},\ \mu \in \mathrm{R},\ k \in \mathrm{T}^d$ operator H Gilbert fazosida o'z-o'ziga qo'shma chegaralangan operator bo'ladi [2-4].

2. $H_{\gamma\mu}(k)$ Fridrixs model operatorining muhim spektri

$H_{\gamma\mu}(k)$ va $H_{0\mu}(k),\ k \in \mathrm{T}^d$ operatorlar ayirmasining rangi ikkiga teng bo'lgani uchun, muhim spektr turg'unligi haqidagi Veyl teoremasiga asosan, $H_{\gamma\mu}(k),\ k \in \mathrm{T}^d$ operatorning muhim spektri $H_{0\mu}(k),\ k \in \mathrm{T}^d$ operatorning muhim spektri $\sigma_{ess}(H_{0\mu}(k))$ bilan ustma-ust tushadi [17], xususan,

$$\sigma_{ess}(H_{\gamma\mu}(k)) = \sigma_{ess}(H_{0\mu}(k)) = \sigma_{ess}(h_\mu(k)) = \sigma(h_0(k)) = [\varepsilon_{\min}(k), \varepsilon_{\max}(k)],$$

tengliklar o'rinli bo'ladi [5-9].

3. $H_{\gamma 0}(k),\ k \in \mathrm{T}^d$ operatorning spektral xossalari

Quyidagi teorema, $d = 1,2$ o'lchamlarda $H_{\gamma 00}(k),\ k \in \mathrm{T}^d$ operatorning xos qiymatlari mavjudligi va unga mos xos funksiyaning analitikligini tavsiflashdan iborat.

Teorema 1. $d = 1,2$ *va* $\gamma > 0\ \mu = 0$ *bo'lsin. U holda ixtiyoriy* $k \in \mathrm{T}^d$ *uchun* $H_{\gamma 0}(k)$ *operator muhim spektrdan tashqarida ikkita* $E^1_{\gamma 0}(k)$ *va* $E^2_{\gamma 0}(k)$ *xos qiymatlarga ega. Va bu xos qiymatlar uchun quyidagi tengsizlik o'rinli*

$$E^1_{\gamma 0}(k) < \varepsilon_{\min}(k) \le \varepsilon_{\max}(k)\varepsilon(k) < E^2_{\gamma 0}(k)$$

Unga mos $f_k = (f_{0,k}, f_{1,k}) \in \mathrm{H}$ xos vektor

$$f^i_{0,k} = \frac{-c\gamma}{\varepsilon(k) - E^i_{\gamma 0}(k)}, \quad f^i_{1,k}(q) = \left(\frac{\gamma^2}{\varepsilon(k) - E^i_{\gamma 0}(k)}\right)\frac{c}{\varepsilon_k(q) - E^i_{\gamma 0}(k)}, i = 1,2$$

ko'rinishga ega, bunda $c \neq 0$ – normalovchi ko'paytuvchi va $f^i_{1,k}(\cdot)$ funksiya T^d da haqiqiy analitik .

Bundan tashqari, $E^i_{\gamma 0}: \mathrm{T}^d \to \mathrm{R},\ k \to E^i_{\gamma 0}(k), i = 1,2$ akslantirish T^d dagi juft va haqiqiy analitik funksiya va $f^i: \mathrm{T}^d \to \mathrm{H},\ k \to f^i_k \in \mathrm{H}$ akslantirish esa, T^d dagi vektor qiymatli analitik akslantirish bo'ladi [10-13].

$a(k,z)$ orqali $\mathrm{T}^d \times (\mathrm{C} \setminus \sigma_{ess}(H_{\gamma 0}(k)))$ da haqiqiy-analitik bo'lgan quyidagi funksiyani belgilaymiz:

$$a(k,z) = \frac{1}{(2\pi)^d} \int_{\mathrm{T}^d} \frac{dq}{z - \varepsilon_k(q)}. \tag{3}$$

Har bir fiksirlangan $k \in T^d$ uchun $C \setminus [\varepsilon_{\min}(k), \varepsilon_{\max}(k)]$ dagi analitik bo'lgan $\Delta_{\gamma 0}(k;\cdot)$ funksiya ($H_{\gamma 0}$ operatorga mos Fredgolm determinanti) ni quyidagicha aniqlaymiz:

$$\Delta_{\gamma 0}(k;z) = 1 - \left[\frac{\gamma^2}{z - \varepsilon(k)} \right] a(k,z). \tag{4}$$

Lemma 1. a) *Ixtiyoriy* $k \in T^d$ *uchun* $a(k,\cdot)$ *funksiya* $(-\infty, \varepsilon_{\min}(k))$ *intervalda manfiy, monoton o'suvchi(mos ravishda* $(\varepsilon_{\max}(k), +\infty)$ *intervalda musbat, monoton kamayuvchi) hamda ixtiyoriy* $z \in C \setminus [\varepsilon_{\min}(k), \varepsilon_{\max}(k)]$ *uchun* T^d *da haqiqiy analitik funksiya bo'ladi.*

b) Ixtiyoriy $\gamma \in R$ $k \in T^d$ uchun

$$\Delta_{\gamma 0}(k;z) = 1 - \left(\frac{\gamma^2}{z - \varepsilon(k)} \right) a(k,z)$$

funksiya $(-\infty, \varepsilon_{\min}(k))$ intervalda monoton o'sadi (mos ravishda $(\varepsilon_{\max}(k), +\infty)$ intervalda monoton kamayadi).

c) Ixtiyoriy $\gamma \in R,$ $k \in T^d$ uchun

$$\Delta^*_{\gamma 0}(k;z) = (z - \varepsilon(k))\Delta_{\gamma 0}(k;z)$$

funksiya $(-\infty, \varepsilon_{\min}(k))$ intervalda ko'pi bilan bitta nolga ega (mos ravishda $(\varepsilon_{\max}(k), +\infty)$ intervalda ko'pi bilan bitta nolga ega).

Isbot. a) $a(k,\cdot)$ ning musbatligi integral ostidagi funksiyaning manfiyligi (mos ravishda musbatligi)hamda Lebeg integralining monotonligidan kelib chiqadi. Shuningdek, $a(k,\cdot)$ ning hosilasi musbat bo'lganligi uchun, u $(-\infty, \varepsilon_{\min}(k))$ da monoton o'suvchi bo'ladi (mos ravishda $a(k,\cdot)$ ning hosilasi manfiy bo'lganligi uchun, u $(\varepsilon_{\max}(k), +\infty,)$ da monoton kamayuvchi bo'ladi) [13-19].

b) 1- lemmaning a) bandiga ko'ra istalgan $z < \varepsilon_{\min}(k)$ uchun

$$\frac{\partial \Delta_{\gamma 0}(k;z)}{\partial z} = \left[\frac{\gamma^2}{(z - \varepsilon(k))^2} a(k,z) \frac{\partial a(k,z)}{\partial z} + \frac{\gamma^2}{(z - \varepsilon(k))} \frac{\partial a(k,z)}{\partial z} \right] > 0$$

tengsizlikning bajarilishi kelib chiqadi, ya'ni $\Delta_{\gamma 0}(k;\cdot)$ funksiya $(-\infty, \varepsilon_{\min}(k))$ intervalda monoton o'sadi (mos ravishda $\Delta_{\gamma 0}(k;\cdot)$ funksiya $(\varepsilon_{\max}(k), +\infty,)$ intervalda monoton kamayadi).

c) Tushunarliki, $\Delta_{\gamma 0}(k;z)$ va $\Delta^*_{\gamma 0}(k;z)$ funksiyalarning $(-\infty, \varepsilon_{\min}(k))$ intervaldagi nollari ustma-ust tushadi (mos ravishda $(\varepsilon_{\max}(k), +\infty,)$ intervaldagi nollari ustma-ust tushadi). $\Delta_{\gamma 0}(k;\cdot)$ ning $(-\infty,\varepsilon_{\min}(k))$ intervalda monoton o'suvchiligidan uning shu intervaldagi nollari soni bittadan oshmaydi (mos ravishda $\Delta_{\gamma 0}(k;\cdot)$ ning $(,\varepsilon_{\max}(k),+\infty)$ intervalda monoton kamayuvchiligidan uning shu intervaldagi nollari soni bittadan oshmaydi). Bu esa $\Delta^{*}_{\gamma 0}(k;\cdot)$ funksiya mazkur intervallardan bittadan ortiq nolga ega emasligini bildiradi [19-23].

Quyidagi lemmada o'z-o'ziga qo'shma $H_{\gamma 0}(k)$ operatorning xos qiymati va $\Delta_{\gamma 0}(k;z)$ funksiyaning noli o'rtasidagi bog'liqlik o'rnatilgan.

Lemma 2. *Ixtiyoriy* $\gamma \neq 0$ *uchun* $z \in \mathrm{C} \setminus [\varepsilon_{min}(k),\varepsilon_{max}(k)]$ *soni* $H_{\gamma 0}(k),\ k \in \mathrm{T}^d$ *operatorning xos qiymati bo'lishi uchun,* $\Delta_{\gamma 0}(k;z)=0$ *tenglikning bajarilishi zarur va yetarli.*

Isbot. Zarurligi. Faraz qilamiz, $z \in \mathrm{C} \setminus \sigma_{ess}(H_{\gamma 0}(k))$ soni $H_{\gamma 0}(k),\quad k \in \mathrm{T}^d$ operatorning xos qiymati hamda $f=(f_0,f_1)\in \mathrm{H}$ unga mos xos vektor bo'lsin. U holda f_0 va f_1 lar quyidagi tenglamalar sistemasini qanoatlantiradi:

$$\begin{cases} (\varepsilon(k)-z)\,f_0=-\gamma(f_1,\alpha_0) \\ (\varepsilon_k(q)-z)\,f_1(q)=-\gamma\alpha_0 f_0. \end{cases} \tag{5}$$

$\sigma_{ess}(H_{\gamma 0}(k))=[\varepsilon_{\min}(k),\varepsilon_{\max}(k)]$ tenglikka ko'ra, ixtiyoriy $z \in \mathrm{C} \setminus [\varepsilon_{\min}(k),\varepsilon_{\max}(k)]$ va $q \in \mathrm{T}^d$ lar uchun

$$\varepsilon_k(q)-z \neq 0, \quad k \in \mathrm{T}^d$$

tengsizlik o'rinli. (5) sistemaning ikkinchi tenglamasidan f_1 uchun quyidagi

$$12\, f_1(q)=\frac{-\gamma\alpha_0 f_0}{\varepsilon_k(q)-z} \tag{6}$$

ifodaga ega bo'lamiz va

$$11c=(f_1,\alpha_0) \tag{7}$$

deb belgilaymiz.

f_1 uchun (6) ifodani (7) tenglikka qo'yib va (5) tenglamalar sistemasini birinchi tenglamasini hisobga olib quyidagi

$$\begin{cases} (\varepsilon(k)-z)\,f_0+\gamma c=0 \\ \gamma a(k,z)\,f_0+c=0 \end{cases} \tag{8}$$

chiziqli tenglamalar sistemasiga kelamiz. Ma'lumki, (3.8) chiziqli tenglamalar sistemasi nolmas (f_0,c) yechimga ega bo'lishi uchun shu sistemaning determinanti

$\Delta_{\gamma 0}(k;z)$ nolga teng bo'lishi yetarli va zarur hamda $\Delta_{\gamma 0}(k;z)=0$ bo'lgan holda bu sistemani yechib, $H_{\gamma 00}$ operatorning xos funksiyasi $f=(f_0,f_1)$ quyidagi ko'rinishga egaligini topamiz:

$$f_0=\frac{-\gamma c}{\varepsilon(k)-z},\quad f_1(q)=\frac{\gamma\alpha_0 f_0}{\varepsilon_k(q)-z} \tag{9}$$

Yetarliligi. Faraz qilamiz, biror $k\in \mathrm{T}^d$ va $z\in \mathrm{C}\setminus[\varepsilon_{\min}(k),\varepsilon_{\max}(k)]$ uchun $\Delta_{\gamma 0}(k;z)=0$ tenglik bajarilsin. U holda (3.9) ko'rinishga ega bo'lgan $f=(f_0,f_1)$ vektor (3.5) tenglamalar sistemasini qanotlantiradi, ya'ni $z\in \mathrm{C}\setminus[\varepsilon_{\min}(k),\varepsilon_{\max}(k)]$ soni $H_{\gamma 0}(k),\ k\in \mathrm{T}^d$ operatorning xos qiymati bo'ladi. Lemma 3.2 isbotlandi.

Lemma 3.

a) $d\ge 1$ va $k\in(-\pi,\pi)$ bo'lsin. U holda yetarlicha kichik va musbat $\varepsilon_{\min}(k)-z$ lar uchun $a(k,z)$ funksiya quyidagi yaqinlashuvchi yoyilmaga ega:

agar $d=2m+2,\ m=0,1,\dots$ bo'lsa,

$$a(k,z)=-\frac{\Phi_0(k)}{2}\left(z-\varepsilon_{\min}(k)\right)^m\ln(\varepsilon_{\min}(k)-z)+$$
$$+\left(z-\varepsilon_{\min}(k)\right)^{m+1}\ln(\varepsilon_{\min}(k)-z)\Phi_{11}(k,z)+\nu(k)+\Phi_2(k,z),$$

agar $d=2m+1,\ m=0,1,\dots$ bo'lsa,

$$a(k,z)=\frac{\pi\Phi_0(k)}{2}\ \frac{\left(z-\varepsilon_{\min}(k)\right)^m}{\sqrt{\varepsilon_{\min}(k)-z}}+$$
$$+\left(\varepsilon_{\min}(k)-z\right)^{m+1/2}\Phi_{12}(k,z)+\nu(k)+\Phi_2(k,z),$$

bunda

$$\Phi_0(k)=\frac{c}{\sqrt{\cos\frac{k_1}{2}\dots\cos\frac{k_d}{2}}},\quad c=const,$$

va

$$\Phi_{12}(k,z)=\sum_{l=0}^{\infty}b_l(k)\left(\varepsilon_{\min}(k)-z\right)^{l/2}$$

hamda $\Phi_{11}(k,\cdot)$ va $\Phi_2(k,\cdot),\ k\in\Pi_{0,\pi}$ funksiyalar $z=\varepsilon_{\min}(k)$ nuqtaning biror $\delta-$ atrofi $V_\delta(\varepsilon_{\min}(k))$ da analitik funksiyalar, $\Phi_2(k,\varepsilon_{\min}(k))=0;\ b_l(k),\ l=0,1,2\dots$ lar esa haqiqiy sonlar

b) $d\ge 1$ va $k\in(-\pi,\pi)$ bo'lsin. U holda yetarlicha kichik va musbat $\varepsilon_{\max}(k)-z$ lar uchun $a(k,z)$ funksiya quyidagi yaqinlashuvchi yoyilmaga ega:

agar $d = 2m+2,\ m = 0,1,\ldots$ bo'lsa,

$$a(k,z) = -\frac{\Phi_0(k)}{2}\left(z-\varepsilon_{\max}(k)\right)^m \ln(z-\varepsilon_{\max}(k)) +$$
$$+\left(z-\varepsilon_{\max}(k)\right)^{m+1}\ln(z-\varepsilon_{\max}(k))\Phi_{11}(k,z)+\nu(k)+\Phi_2(k,z),$$

agar $d = 2m+1,\ m = 0,1,\ldots$ bo'lsa,

$$a(k,z) = \frac{\pi\Phi_0(k)}{2}\ \frac{\left(z-\varepsilon_{\max}(k)\right)^m}{\sqrt{z-\varepsilon_{\max}(k)}} +$$
$$+\left(z-\varepsilon_{\max}(k)\right)^{m+1/2}\Phi_{12}(k,z)+\nu(k)+\Phi_2(k,z),$$

bunda

$$\Phi_0(k) = \frac{c}{\sqrt{\cos\frac{k_1}{2}\ldots\cos\frac{k_d}{2}}},\quad c = const,$$

va

$$\Phi_{12}(k,z) = \sum_{l=0}^{\infty} b_l(k)\left(z-\varepsilon_{\max}(k)\right)^{l/2}$$

hamda $\Phi_{11}(k,\cdot)$ va $\Phi_2(k,\cdot),\ k\in\Pi_{0,\pi}$ funksiyalar $z=\varepsilon_{\max}(k)$ nuqtaning biror $\delta-$ atrofi $V_\delta(\varepsilon_{\max}(k))$ da analitik funksiyalar, $\Phi_2(k,\varepsilon_{\max}(k))=0;\ b_l(k),\ l=0,1,2\ldots$ lar esa haqiqiy sonlar.

Isbot. Lemmaning a) va b) tasdig'larii [11] dagi 4-lemma kabi isbotlanadi.

$z<\varepsilon_{\min}(k)$ bo'lsin. U holda 3-lemma va (4) tenglikka asosan fiksirlangan $\gamma>0$ larda

$$\lim_{z\to\varepsilon_{\min}(k)-}\Delta_{\gamma 0}(k;z) = -\infty,\qquad \lim_{z\to-\infty}\Delta_{\gamma 0}(k;z)=1.$$

tengliklar o'rinli. $\Delta_{\gamma 0}(k;\cdot)$ funksiya $(-\infty,\varepsilon_{\min}(k))$ intervalda uzluksiz, monoton o'suvchi va ishorasi o'zgarganligi sababli $\Delta_{\gamma 0}(k;z)=0$ tenglama mazkur intervalda yagona $z=E^1_{\gamma 0}(k)$ yechimga ega. Natijada 1 va 2 - lemmalarga ko'ra $H_{\gamma 0}(k)$ operator yagona $(-\infty,\varepsilon_{\min}(k))$ intervalda $E^1_{\gamma 0}(k)<\varepsilon_{\min}(k)$ xos qiymatga ega.

$E^1_{\gamma 0}(k)$ ning T^d da haqiqiy analitikligi u $\Delta_{\gamma 0}(k;z)=0$ tenglamaning yagona yechimi ekanligidan hamda $\Delta_{\gamma 0}(k;\cdot)$ va $\Delta_{\gamma 0}(\cdot;z)$ funksiyalarning mos ravishda $\mathrm{C}\setminus[\varepsilon_{\min}(k),\varepsilon_{\max}(k)]$ va T^d da haqiqiy-analitik ekanligidan kelib chiqadi. Shuningdek, ixtiyoriy $k\in\mathrm{T}^d$ uchun

$$0=\Delta_{\gamma 0}(-k,E^1_{\gamma 0}(-k))=\Delta_{\gamma 0}(k,E^1_{\gamma 0}(-k))=\Delta_{\gamma 0}(k,E^1_{\gamma 0}(k))$$

tengliklar bajarilishini hisobga olsak, $\Delta_{\gamma 0}(k;\cdot)$ tenglamaning yechimi yagonaligidan $E^1_{\gamma 0}(k)=E^1_{\gamma 0}(-k)$ tenglikning bajarilishi kelib chiqadi. Shuning uchun $E^1_{\gamma 0}(k)$ funksiya T^d dagi juft funksiya bo'ladi [23-26]..

Shuni ta'kidlaymizki, $E^1_{\gamma 0}(k)\in \mathrm{C}\setminus[\varepsilon_{\min}(k),\varepsilon_{\max}(k)]$ soni $H_{\gamma 0}(k),$ $k\in\mathrm{T}^d$ operatorning xos qiymati bo'lishi uchun

$$\begin{cases}(\varepsilon(k)-E^1_{\gamma 0}(k))f_0=-\gamma(f_1,\alpha_0)\\ (\varepsilon_k(q)-E^1_{\gamma 0}(k))f_1(q)=-\gamma\alpha_0 f_0.\end{cases} \tag{10}$$

tenglamalar sistemasi notrivial $f_k=(f_{0,k},f_{1,k})\in\mathrm{H}$ yechimga ega bo'lishi zarur va yetarli, bunda

$$f_{0,k}=\frac{-\gamma c}{\varepsilon(k)-E^1_{\gamma 0}(k)},\quad f_{1,k}(q)=\left(\frac{\gamma^2}{\varepsilon(k)-E^1_{\gamma 0}(k)}+\mu\right)\frac{c}{\varepsilon_k(q)-E^1_{\gamma 0}(k)}, \tag{11}$$

va $c\neq 0$ - normalovchi ko'paytuvchi. $\varepsilon_k(\cdot)$ funksiyaaning T^d da analitikligidan va $\varepsilon(k)-E^1_{\gamma 0}(k)>0$ ekanligidan $f_{1,k}(\cdot)$ ning T^d da analitikligi kelib chiqadi. $f_{0,k}$ va $f_{1,k}$, lar uchun (3.11) ifodalardan va $E^1_{\gamma 0}(\cdot)$ funksiyaning T^d da analitik ekanligidan xulosa qilish mumkinki, $f:\mathrm{T}^d\to\mathrm{H},\ \ k\to f_k\in\mathrm{H}$ akslantirish T^d dagi analitik akslantirish bo'ladi. $f:k\to f_k\in\mathrm{H}$ akslantirishning analitikligi f_k vektornig $f_{0,k}$ va $f_{1,k}$ komponentalari analitik ekanligidan kelib chiqadi.

$z>\varepsilon_{\max}(k)$ bo'lsin. U holda 3 - lemma va (4) tenglikka asosan fiksirlangan $\gamma>0$ larda

$$\lim_{z\to\varepsilon_{\max}(k)+}\Delta_{\gamma 0}(k;z)=-\infty,\qquad \lim_{z\to+\infty}\Delta_{\gamma 0}(k;z)=1.$$

tengliklar o'rinli. $\Delta_{\gamma 0}(k;\cdot)$ funksiya $(\varepsilon_{\max}(k),+\infty)$ intervalda uzluksiz, monoton o'suvchi va ishorasi o'zgarganligi sababli $\Delta_{\gamma 0}(k;z)=0$ tenglama mazkur intervalda yagona $z=E^2_{\gamma 0}(k)$ yechimga ega. Natijada 1 va 2 lemmalarga ko'ra $H_{\gamma 0}(k)$ operator yagona $(\varepsilon_{\max}(k),+\infty)$ intervalda $E^2_{\gamma 0}(k)>\varepsilon_{\max}(k)$ xos qiymatga ega. Xuddi shunday bu xos qiymat uchun ham yuqoridagi mulohazalar o'rinli.

4. $H_{\gamma\mu}(k),k\in\mathrm{T}^d$ operatorning xos qiymatlari haqida

Quyidagi teorema, $d=1,2$ o'lchamlarda $H_{\gamma\mu}(k),$ $k\in\mathrm{T}^d$ operatorning xos qiymatlari mavjudligi haqida .

Teorema 2. a) $d=1,2$ *va* $\gamma \neq 0\ \mu \geq 0$ *bo'lsin. U holda ixtiyoriy* $k \in \mathrm{T}^d$ *uchun* $H_{\gamma\mu}(k)$ *operator muhim spektrdan tashqarida ikkita* $E_{\gamma\mu}^1(k)$ *va* $E_{\gamma\mu}^2(k)$ *xos qiymatlarga ega. Va bu xos qiymatlar uchun quyidagi tengsizlik o'rinli*

$$E_{\gamma\mu}^1(k) < \varepsilon_{\min}(k) \leq \varepsilon_{\max}(k) < \varepsilon(k) < E_{\gamma\mu}^2(k)$$

b) $d=1,2$ va $\gamma \neq 0\ \mu \geq 0$ bo'lsin. U holda ixtiyoriy $k \in \mathrm{T}^d$ uchun $H_{\gamma\mu}(k)$ operator $(\varepsilon_{\max}(k), \varepsilon(k))$ intervalda xos qiymatga ega emas.

c) $d=1,2$ va $\gamma = 0\ \mu > 0$ bo'lsin. U holda ixtiyoriy $k \in \mathrm{T}^d$ uchun $H_{\gamma\mu}(k)$ operator muhim spektrdan tashqarida ikkita $E_{\gamma\mu}^1(k)$ va $E_{\gamma\mu}^2(k)$ xos qiymatlarga ega. Va bu xos qiymatlar uchun quyidagi tengsizlik o'rinli

$$E_{\gamma\mu}^1(k) < \varepsilon_{\min}(k) \leq \varepsilon_{\max}(k) < \varepsilon(k) = E_{\gamma\mu}^2(k)$$

Teorema 3. *Faraz qilamizki* $d=1,2$ *va* $\gamma \neq 0\ \mu \leq 0$ *bo'lsin.*

a) $\dfrac{\gamma^2}{\mu} > \varepsilon_{\max}(k) - \varepsilon(k)$ va $k \in \mathrm{T}^d$ bo'lsin. U holda $H_{\gamma\mu}(k)$ operator muhim spektrdan tashqarida ikkita $E_{\gamma\mu}^1(k)$ va $E_{\gamma\mu}^2(k)$ xos qiymatlarga ega. Va bu xos qiymatlar uchun quyidagi tengsizlik o'rinli

$$\varepsilon_{\max}(k) < E_{\gamma\mu}^1(k) < \varepsilon(k) < E_{\gamma\mu}^2(k)$$

b) $\varepsilon_{\max}(k) - \varepsilon(k) \geq \dfrac{\gamma^2}{\mu} \geq \varepsilon_{\min}(k) - \varepsilon(k)$ va $k \in \mathrm{T}^d$ bo'lsin.U holda $H_{\gamma\mu}(k)$ operator yagona $E_{\gamma\mu}^2(k)$ xos qiymatga ega va bu xos qiymat quyidagi tengsizlikni qanoatlantiradi

$$\varepsilon(k) < E_{\gamma\mu}^2(k)$$

c) $\dfrac{\gamma^2}{\mu} < \varepsilon_{\min}(k) - \varepsilon(k)$ va $k \in \mathrm{T}^d$ bo'lsin. U holda $H_{\gamma\mu}(k)$ operator muhim spektrdan tashqarida ikkita $E_{\gamma\mu}^1(k)$ va $E_{\gamma\mu}^2(k)$ xos qiymatlarga ega. Va bu xos qiymatlar uchun quyidagi tengsizlik o'rinli

$$E_{\gamma\mu}^1(k) < \varepsilon_{\min}(k) < \varepsilon(k) < E_{\gamma\mu}^2(k)$$

c) $d=1,2$ va $\gamma = 0\ \mu < 0$ bo'lsin. U holda $H_{\gamma\mu}(k)$ operator muhim spektrdan tashqarida ikkita $E_{\gamma\mu}^1(k) = \varepsilon(k)$ va $E_{\gamma\mu}^2(k)$ xos qiymatlarga ega. Va bu xos qiymatlar uchun quyidagi tengsizlik o'rinli

$$\varepsilon_{\max}(k) < E_{\gamma\mu}^1(k) = \varepsilon(k) < \varepsilon(k) < E_{\gamma\mu}^2(k)$$

$a(k,z)$ orqali $\mathrm{T}^d \times (\mathrm{C} \setminus \sigma_{ess}(H_{\gamma\mu}(k)))$ da haqiqiy-analitik bo'lgan quyidagi funksiyani belgilaymiz:

$$a(k,z) = \frac{1}{(2\pi)^d} \int_{\mathrm{T}^d} \frac{dq}{z - \varepsilon_k(q)}. \tag{12}$$

Har bir fiksirlangan $k \in \mathrm{T}^d$ uchun $\mathrm{C} \setminus [\varepsilon_{\min}(k), \varepsilon_{\max}(k)]$ dagi analitik bo'lgan $\Delta_{\gamma\mu}(k;\cdot)$ funksiya ($H_{\gamma 0}$ operatorga mos Fredgolm determinanti) ni quyidagicha aniqlaymiz:

$$\Delta_{\gamma\mu}(k;z) = 1 - \left[\frac{\gamma^2}{z - \varepsilon(k)} - \mu\right] a(k,z). \tag{13}$$

Lemma 4. a) *Ixtiyoriy* $k \in \mathrm{T}^d$ *uchun* $a(k,\cdot)$ *funksiya* $(-\infty, \varepsilon_{\min}(k))$ *intervalda manfiy, monoton o'suvchi(mos ravishda* $(\varepsilon_{\max}(k), +\infty)$ *intervalda musbat, monoton kamayuvchi) hamda ixtiyoriy* $z \in \mathrm{C} \setminus [\varepsilon_{\min}(k), \varepsilon_{\max}(k)]$ *uchun* T^d *da haqiqiy analitik funksiya bo'ladi.*

b) Ixtiyoriy $\gamma \in \mathrm{R}$ $k \in \mathrm{T}^d$ uchun

$$\Delta_{\gamma\mu}(k;z) = 1 - \left(\frac{\gamma^2}{z - \varepsilon(k)} - \mu\right) a(k,z)$$

funksiya $(-\infty, \varepsilon_{\min}(k))$ intervalda monoton o'sadi (mos ravishda $(\varepsilon_{\max}(k), +\infty)$ intervalda monoton kamayadi).

c) Ixtiyoriy $\gamma \in \mathrm{R}$, $k \in \mathrm{T}^d$ uchun

$$\Delta^*_{\gamma\mu}(k;z) = (z - \varepsilon(k))\Delta_{\gamma\mu}(k;z)$$

funksiya $(-\infty, \varepsilon_{\min}(k))$ intervalda ko'pi bilan bitta nolga ega (mos ravishda $(\varepsilon_{\max}(k), +\infty)$ intervalda ko'pi bilan bitta nolga ega) [27-32].

Isbot. Lemmaning isboti 1-lemma isboti kabi isbotlanadi.

Quyidagi lemmada o'z-o'ziga qo'shma $H_{\gamma\mu}(k)$ operatorning xos qiymati va $\Delta_{\gamma\mu}(k;z)$ funksiyaning noli o'rtasidagi bog'liqlik o'rnatilgan.

Lemma 5. *Ixtiyoriy* $\gamma \in \mathrm{R}$ *va* $\mu \le 0$ *uchun* $z \in \mathrm{C} \setminus [\varepsilon_{min}(k), \varepsilon_{max}(k)]$ *soni* $H_{\gamma\mu}(k),\ k \in \mathrm{T}^d$ *operatorning xos qiymati bo'lishi uchun,* $\Delta_{\gamma\mu}(k;z) = 0$ *tenglikning bajarilishi zarur va yetarli.*

Isbot. Lemmaning isboti 2-lemma isboti kabi isbotlanadi.

2- teoremaning isboti**.** Teoremaning a) bandining sharti bajarilsin . U holda

3- lemma va (13) tenglikka asosan

$$\lim_{z\to\varepsilon(k)-}\Delta_{\gamma 0}(k;z)=-\infty,\qquad \lim_{z\to+\infty}\Delta_{\gamma 0}(k;z)=1.$$

$$\lim_{z\to\varepsilon_{\min}(k)-}\Delta_{\gamma 0}(k;z)=-\infty,\qquad \lim_{z\to-\infty}\Delta_{\gamma 0}(k;z)=1.$$

tengliklar o'rinli. $\Delta_{\gamma\mu}(k;\cdot)$ funksiya $(-\infty,\varepsilon_{\min}(k))$ intervalda uzluksiz, monoton o'suvchi (mos ravishfa $(\varepsilon_{\max}(k),+\infty)$ intervalda uzluksiz, monoton kamayuvchi)va ishorasi o'zgarganligi sababli $\Delta_{\gamma\mu}(k;z)=0$ tenglama mazkur intervalda yagona $z=E^1_{\gamma\mu}(k)$ va $z=E^2_{\gamma\mu}(k)$ yechimlarga ega. Natijada 1 va 2- lemmalarga ko'ra $H_{\gamma\mu}(k)$ operator muhim spektrdan tashqarida ikkita $E^1_{\gamma\mu}(k)$ va $E^2_{\gamma\mu}(k)$ xos qiymatlarga ega. Va bu xos qiymatlar uchun quyidagi tengsizlik o'rinli

$$\varepsilon_{\max}(k)<E^1_{\gamma\mu}(k)<\varepsilon(k)<E^2_{\gamma\mu}(k).$$

Xuddi shunday teremaning qolgan bandlari uchun yuqoridagi mulohazalar o'rinli [33-38].

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