Bir o'lchamli panjaradagi bir zarrachali Shryodinger operatorining xos qiymatlari soni

Nargiza Ahmedovna Tosheva
Fotima Muhiddin qizi Sayfullayeva
Buxoro davlat universiteti

Annotatsiya: Ushbu maqolada $l^2(Z^d)$ fazoda chegaralangan o'z-o'ziga qo'shma operator qaralgan. Panjarada qaralgan operatorning xos qiymati topilgan. Undan tashqari bu operator uchun Birman-Shvinger prinsipi keltirilgan.

Kalit so'zlar: Hilbert fazosi, chiziqli operator, chegaralangan operator, o'z-o'ziga qo'shma fazolar, invariant qism fazo, Veyl teoremasi, uzluksiz spektr

Number of eigenvalues of a single-particle Schroedinger orator in a one-dimensional range

Nargiza Ahmedovna Tosheva
Fotima Muhiddin qizi Sayfullayeva
Bukhara State University

Abstract: In this article A bounded self-adjoint operator is considered in $l^2(Z^d)$ space. In addition, the Birman-Schwinger principle is given for this operator. The eigenvalue of the operator viewed in the grid is found.

Keywords: Hilbert space, linear operator, bounded operator, self-adjoint operator, invariant subspace, Weyl's theorem, continuous spectrum

$T^d - d -$ o'lchamli tor, ya'ni $(-\pi,\pi]^d -$ mos qarama-qarshi tomonlari aynan teng bo'lgan kub bo'lsin. Ta'kidlab o'tish joizki, $T^d \equiv (-\pi,\pi]^d \subset P^d$ to'plamdagi qo'shish va haqiqiy songa ko'paytirish amallari P^d dagi $(2\pi Z^1)^d$ modul bo'yicha amallar sifatida tushuniladi. rw$\ell^2(Z^d) - d -$o'lchamli butun sonli panjara Z^dda aniqlangan kvadrati bilan jamlanuvchi funksiyalarning Hilbert fazosi bo'lsin [1-7].

Koordinatali tasvirida Z^d panjarada harakatlanuvchi bir kvant zarrachaning erkin Hamiltoniani $l^2(Z^d)$ fazoda chegaralangan o'z-o'ziga qo'shma operator orqali quyidagi fopmula bo'yicha aniqlanadi:

$$(\hat{h}_0\hat{\varphi})(x) = \sum_{s\in Z^d}\hat{\varepsilon}(x-s)\hat{\varphi}(s), \quad \hat{\varphi}\in \ell^2(Z^d).$$

Bu yerda $\hat{\varepsilon}(\cdot)$ funksiya Z^d da aniqlangan va quyidagi ko'rinishga ega

$$\hat{\varepsilon}(s) = \begin{cases} d, & \text{agar} \quad |s| = 0, \\ -\frac{1}{2}, & \text{agar} \quad |s| = 1, \\ 0, & \text{agar} \quad |s| > 1, \end{cases}$$

$s = (s^{(1)},\ldots,s^{(d)}) \in Z^d, \qquad |s| = |s^{(1)}| + \cdots + |s^{(d)}|$.

Koopdinatali tasvirda $\hat{v}_{\mu\lambda}$ potensial maydondagi bir zarrachaning to'la Hamiltoniani $\hat{h}_0$ erkin Hamiltonianning chegaralangan qo'zg'alishi sifatida quyidagicha aniqlanadi:

$$\hat{h}_{\mu\lambda} = \hat{h}_0 - \hat{v}_{\mu\lambda}.$$

bu yerda $\hat{v}_{\mu\lambda}$ $l^2(Z^d)$ fazoda $\hat{v}_{\mu\lambda}(\cdot)$ funksiyaga ko'paytirish operatori, ya'ni

$$(\hat{v}_{\mu\lambda}\hat{\varphi})(x) = \hat{v}_{\mu\lambda}(x)\hat{\varphi}(x), \quad \hat{\varphi}\in \ell^2(Z^d).$$

$\hat{v}_{\mu\lambda}(\cdot)$ funksiya Z^d da quyidagicha aniqlangan

$$\hat{v}_{\mu\lambda}(s) = \begin{cases} \mu, & \text{agar} \quad |s| = 0, \\ \frac{\lambda}{2}, & \text{agar} \quad |s| = 1, \\ 0, & \text{agar} \quad |s| > 1, \end{cases}$$

bunda $\mu \geq 0$ va $\lambda \geq 0$ bir vaqtda nolga teng bo'lmagan sonlar.

Ta'kidlash joizki, $\hat{h}_{\mu\lambda}$ $l^2(Z^d)$ gilbert fazosida chegaralangan o'z-o'ziga qo'shma operator bo'ladi.

$l_e^2(Z^d) \subset l^2(Z^d) -$ wZ^d dagi juft funksiyalar qism fazosi bo'lsin. Eslatib o'tish joizki, $l_e^2(Z^d)$ Hilbert fazosi $\hat{h}_{\mu\lambda}$ operator ta'siriga nisbatan invariant qism fazo bo'ladi. Shuning uchun quyida $\hat{h}_{\mu\lambda}$ operatorning $l_e^2(Z^d)$ fazodagi qismi $\hat{h}_{\mu\lambda}/l_e^2(Z^d)$ni ham $\hat{h}_{\mu\lambda}$ orqali belgilaymiz.

$v_{\mu\lambda}-$ rangi uchыdan oshmaydigan integral operator bo'lganligi uchun Veyl teoremasiga ko'ra, $h_{\mu\lambda}$ operatorning uzluksiz spektri $\sigma_{cont}(h_{\mu\lambda})$, $\mu,\lambda \geq 0$ lardan bog'liqsiz va $\sigma(h_0)$ operatorning spektri $\sigma(h_0)$ bilan ustma-ust tushadi. Shunday qilib, quyidagi tengliklar o'rinli

$$\sigma_{cont}(h_{\mu\lambda}) = \sigma(h_0) = [0, 2d].$$

$L_e^2(\mathrm{T}^d)$ da quyidagi ortonormal sistemani qaraymiz:

$$\alpha_0 = \frac{1}{(2\pi)^{\frac{d}{2}}}, \qquad \alpha_i(p) = \frac{\sqrt{2}}{(2\pi)^{\frac{d}{2}}}\cos p^{(i)}, \quad i = \overline{1,d}.$$

$v_{\mu\lambda}$ operator quyidagi ko'rinishda tasvirlanadi:

$$v_{\mu\lambda} f = \mu\alpha_0(f,\alpha_0) + \frac{\lambda}{2}\sum_{i=1}^{d}(f,\alpha_i)\alpha_i,$$

bunda $(\cdot,\cdot)-$ $L_e^2(\mathrm{T}^d)$ dagi skalyar ko'paytma.

1-lemma. $v_{\mu\lambda}$ nomanfiy operator, ya'ni ixtiyoriy $f \in L_e^2(\mathrm{T}^d)$ uchun $(v_{\mu\lambda} f, f) \geq 0$ tengsizlik o'rinli.

$v_{\mu\lambda} \geq 0$ operatorning nomanfiyligidan, uning $v_{\mu\lambda}^{\frac{1}{2}} \geq 0$ kvadrat ildizi mavjud. $v_{\mu\lambda}^{\frac{1}{2}}$ operator $L_e^2(\mathrm{T}^d)$ fazoda quyidagi formula bo'yicha aniqlanadi

$$(v_{\mu\lambda}^{\frac{1}{2}} f)(p) = (2\pi)^{-\frac{d}{2}} \int_{\mathrm{T}^d} v_{\mu\lambda}^{\frac{1}{2}}(p-q) f(q)dq,$$

bunda

$$v_{\mu\lambda}^{\frac{1}{2}}(p) = (2\pi)^{-\frac{d}{2}} \sum_{s\in \mathrm{Z}^d} \hat{v}_{\mu\lambda}^{\frac{1}{2}}(s) e^{i(p,s)}$$

va $\hat{v}_{\mu\lambda}^{\frac{1}{2}}(\cdot)$ funsiya $\hat{v}_{\mu\lambda}(\cdot)$ funksiyaning musbat kvadrat ildizi.

$v_{\mu\lambda}$ integral operatorining aniqlanishidan uning kvadrat ildizi $v_{\mu\lambda}^{\frac{1}{2}}$ quyidagicha aniqlanadi

$$v_{\mu\lambda}^{\frac{1}{2}} f = \sqrt{\mu}\alpha_0(f,\alpha_0) + \sqrt{\frac{\lambda}{2}}\sum_{i=1}^{d}(f,\alpha_i)\alpha_i. \tag{1}$$

$\mathrm{X}-$ kompleks tekislik va $r_0(z)$, $z \in \mathrm{X}\setminus[0,2d]-$ h_0 operatorning rezolventasi bo'lsin.

$\varepsilon(q) = \varepsilon(q^{(1)},\ldots,q^{(d)}) = \sum_{i=1}^{d}(1-\cos q^{(i)})$ funksiya $q^{(i)}$ va $q^{(j)}$ $i,j = \overline{1,d}$ o'zgaruvchilarning o'rnini almashtirishga nisbatan simmetrik funksiya bo'lganligi uchun ushbu

$$\int_{\mathrm{T}^d} \frac{\cos q^{(i)} dq}{\varepsilon(q)-z}, \quad \int_{\mathrm{T}^d} \frac{\cos^2 q^{(i)} dq}{\varepsilon(q)-z}, \quad \text{и} \quad \int_{\mathrm{T}^d} \frac{\cos q^{(i)} \cos q^{(j)} dq}{\varepsilon(q)-z}$$

integrallar $i,j = \overline{1,d}, \quad i \neq j$ lardan bog'liq emas [7-12].

Quyidagi belgilashlarni kiritamiz

$$a(z)=(\alpha_0,r_0(z)\alpha_0)=\frac{1}{(2\pi)^d}\int_{\mathrm{T}^d}\frac{dq}{\varepsilon(q)-z},$$

$$b(z)=(\alpha_0,r_0(z)\alpha_i)=\frac{\sqrt{2}}{(2\pi)^d}\int_{\mathrm{T}^d}\frac{\cos q^{(i)}dq}{\varepsilon(q)-z},$$

$$c(z)=(\alpha_i,r_0(z)\alpha_i)=\frac{2}{(2\pi)^d}\int_{\mathrm{T}^d}\frac{\cos^2 q^{(i)}dq}{\varepsilon(q)-z},$$

$$d(z)=(\alpha_i,r_0(z)\alpha_j)=\frac{2}{(2\pi)^d}\int_{\mathrm{T}^d}\frac{\cos q^{(i)}\cos q^{(j)}dq}{\varepsilon(q)-z}, \tag{2}$$

$z<0,\quad i,j=\overline{1,d}, i\neq j$.

Ixtiyoriy fiksirlangan $\mu,\lambda\geq 0$ va $z\in \mathrm{X}\setminus[0,2d]$ lar uchun $L_e^2(\mathrm{T}^d)$ fazoda quyidagi formula bilan ta'sir qiluvchi chekli o'lchamli Birman–Shvinger integral operatori $G_{\mu\lambda}(z)$ ni aniqlaymiz

$$G_{\mu\lambda}(z)=v_{\mu\lambda}^{\frac{1}{2}}r_0(z)v_{\mu\lambda}^{\frac{1}{2}}.$$

$v_{\mu\lambda}^{\frac{1}{2}}$ operatorning (1) tenglik bilan aniqlanishiga ko'ra, $G_{\mu\lambda}(z)$ operator quyidagi ko'rinishda tasvirlanadi

$$G_{\mu\lambda}(z)f=\left(\mu a(z)(f,\alpha_0)+\sqrt{\frac{\mu\lambda}{2}}b(z)\sum_{i=1}^{d}(f,\alpha_i)\right)\alpha_0+$$

$$+\sum_{i=1}^{d}[\sqrt{\frac{\mu\lambda}{2}}b(z)(f,\alpha_0)+\frac{\lambda}{2}c(z)(f,\alpha_i)+\frac{\lambda}{2}d(z)\sum_{i\neq j=1}^{d}(f,\alpha_j)]\alpha_i. \tag{3}$$

(3) tenglikdan $G_{\mu\lambda}(z)$ operatorning rangi $z\in \mathrm{X}\setminus[0,2d]$ dan bog'liqmas va $d+1$ dan oshmaydi.

Ixtiyoriy fiksirlangan $\mu,\lambda\geq 0$ uchun $h_{\mu\lambda}-zI$ operatorning determinantini $I-G_{\mu\lambda}(z)$ operatorning Fredgolm determinanti kabi aniqlaymiz

$$\Delta(\mu,\lambda;z):=det(h_{\mu\lambda}-zI):=det(I-G_{\mu\lambda}(z)). \tag{4}$$

Ravshanki, ixtiyoriy $\mu,\lambda\geq 0$ uchun $\Delta(\mu,\lambda;\cdot)$ funksiya $\mathrm{X}\setminus[0,2d]$ sohada analitik bo'ladi.

2-lemma. Barcha $\mu,\lambda\geq 0$ va $z\in \mathrm{X}\setminus[0,2d]$ uchun quyidagi tengliklar o'rinli

$$\Delta_d(\mu,\lambda;z)=\Delta_d^{(1)}(\mu,\lambda;z)(\Delta_d^{(22)}(\lambda;z))^{d-1},$$

$$\Delta_d(\mu,0;z)=1-\mu a(z),\quad \Delta_d(0,\lambda;z)=\Delta_d^{(21)}(\lambda;z)(\Delta_d^{(22)}(\lambda;z))^{d-1},$$

bunda

$$\Delta_d^{(1)}(\mu,\lambda;z)=\Delta_d(\mu,0;z)\Delta_d^{(21)}(\lambda;z)-\frac{d\mu\lambda}{2}b^2(z),$$

$$\Delta_d^{(21)}(\lambda;z)=1-\frac{\lambda}{2}(c(z)+(d-1)d(z)),\quad \Delta_d^{(22)}(\lambda;z)=1-\frac{\lambda}{2}(c(z)-d(z)),$$

$d=1,2,3.$

1-lemma isboti. Haqiqatan ham, ixtiyoriy $f\in L_e^2(\mathrm{T}^d)$ uchun quyidagiga ega bo'lamiz

$$(v_{\mu\lambda}f,f)=\int_{\mathrm{T}^d}(v_{\mu\lambda}f)(p)\overline{f(p)}dp=\frac{1}{(2\pi)^d}\int_{\mathrm{T}^d}[\int_{\mathrm{T}^d}(\mu+\lambda\sum_{i=1}^d\cos p^{(i)}\cos t^{(i)})f(t)dt]\overline{f(p)}dp=$$

$$=\frac{1}{(2\pi)^d}[\mu\int_{\mathrm{T}^d}f(t)dt\overline{\int_{\mathrm{T}^d}f(p)dp}+\lambda\sum_{i=1}^d\int_{\mathrm{T}^d}\cos t^{(i)}f(t)dt\overline{\int_{\mathrm{T}^d}\cos p^{(i)}f(p)dp}]=$$

$$=\frac{1}{(2\pi)^d}[\mu|\int_{\mathrm{T}^d}f(t)dt|^2+\lambda\sum_{i=1}^d|\int_{\mathrm{T}^d}\cos t^{(i)}f(t)dt|^2]\geq 0.$$

$L_{d+1}\subset L_e^2(\mathrm{T}^d)-$ orqali 1 va $\cos p^{(i)}\ i=\overline{1,d}$ funksiyalarga tortilgan $d+1-$ o'lchamli qism fazoni belgilaymiz.

1-eslatma. $v_{\mu\lambda}$ operator $L_e^2(\mathrm{T}^d)$ Hilbert fazoni L_{d+1} qism fazoga akslantiradi.

2-eslatma. $v_{\mu\lambda}-$ operatorning $L_e^2(\mathrm{T}^d)$ fazoda nomanfiyligi va $\sup\limits_{f\neq 0}(h_{\mu\lambda}f,f)\leq\sup\limits_{f\neq 0}(h_0f,f)$ tengsizlikdan $h_{\mu\lambda}$ operatoruzluksiz spektr $[0,2d]$ dan o'ngda yotuvchi xos qiymatga ega emas.

3-lemma. Ixtiyoriy $\mu,\lambda\geq 0$ uchun $z<0$ soni $h_{\mu\lambda}$ operatorning xos qiymati bo'lishi uchun 1 soni $G_{\mu\lambda}(z)$ operatorning xos qiymati bo'lishi zarur va yetarli.

Isboti. $z<0$ soni $h_{\mu\lambda}$ operatorning xos qiymati va $f\in L_e^2(\mathrm{T}^d)$ unga mos xos funksiya bo'lsin, ya'ni

$$h_{\mu\lambda}f=zf\qquad\text{yoki}\qquad(h_0-z)f=v_{\mu\lambda}f.$$

Bu yerdan

$$f=r_0(z)v_{\mu\lambda}f$$

tenglikni hosil qilamiz. Bundan esa

$$v_{\mu\lambda}^{\frac{1}{2}}f=(v_{\mu\lambda}^{\frac{1}{2}}r_0(z)v_{\mu\lambda}^{\frac{1}{2}})v_{\mu\lambda}^{\frac{1}{2}}f=G_{\mu\lambda}(z)v_{\mu\lambda}^{\frac{1}{2}}f$$

tenglikka ega bo'lamiz.

Teskarisini faraz qilaylik. 1 soni $G_{\mu\lambda}(z)$ operatorning xos qiymati va $\varphi\in L_e^2(\mathrm{T}^d)$ unga mos xos funksiya bo'lsin, ya'ni

$$\varphi=(v_{\mu\lambda}^{\frac{1}{2}}r_0(z)v_{\mu\lambda}^{\frac{1}{2}})\varphi.$$

Bu yerdan

$$\psi = v_{\mu\lambda} r_0(z)\psi$$

tenglik o'rinli bo'ladi, bunda $\psi = v_{\mu\lambda}^{\frac{1}{2}}\varphi$. $f = r_0(z)\psi$ belgilash orqali

$$(h_0 - z)f = v_{\mu\lambda} f$$

tenglikka ega bo'lamiz, ya'ni $f-$ ushbu $h_{\mu\lambda}$ operatorning $z<0$ xos qiymatiga mos xos funksiyasi bo'ladi.

H Hilbert fazosida aniqlangan va $\beta \in \mathrm{P}$ nuqtadan o'ngda (mos holda chapda) muhim spektrga ega bo'lmagan A chegaralangan o'z-o'ziga qo'shma operator uchun $n_+(\beta, A)$ (mos holda $n_-(\beta, A)$) sonni quyidagicha aniqlaymiz

$$n_+(\beta, A) = \sup\{\dim L:\ L \subset H;\ (Af, f) > \beta,\ \|f\| = 1\}$$

$$\left(n_-(\beta, A) = \sup\{\dim L:\ L \subset H;\ (Af, f) < \beta,\ \|f\| = 1\}\right).$$

$n_+(\beta, A)$ (mos holda $n_-(\beta, A)$) soni A operatorning β dan o'ngda (mos holda chapda) yotuvchi xos qiymatlari soniga mos keladi.

Xos qiymatlar muammosini kamaytirish Birman [13] va Shvinger [14-15] tomonidan umumqabul qilingan bir jinsli Lipman–Shvinger tenglamasi ko'paytmasiga keltirilgan.

4-lemma. (Birman–Shvinger prinsipi). Ixtiyoriy $\mu, \lambda \geq 0$ va $z \leq 0$ uchun

$$n_-(z, h_{\mu\lambda}) = n_+(1, G_{\mu\lambda}(z))$$

tenglik o'rinli.

O'z-o'ziga qo'shma $h_{\mu\lambda}$ operatorining xos qiymatlari va Fredholm determinanti $\Delta(\mu, \lambda; z)$ nollari o'rtasidagi bog'liqlik quyidagi lemma bilan o'rnatiladi

5-lemma. Ixtiyoriy $\mu, \lambda \geq 0$ uchun $z \in \mathrm{X} \setminus [0, 2d]$ soni $h_{\mu\lambda}$ operatorning $m-$ karrali xos qiymati bo'lishi uchun u $\Delta(\mu, \lambda; z)$ funksiyaning $m-$ karrali noli bo'lishi zarur va yetarli.

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