Sturm-Liouville problem with general inverse symmetric potential

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Abstract: For an inverse nonselfadjoint Sturm-Liouville problem with a symmetric potential and general boundary conditions, the uniqueness theorems are established and proven. Six eigenvalues and a spectrum are the spectral information utilized for the original reconstruction of Sturm-Liouville problems. Additionally, it is established that an inverse self-adjoint Sturm-Liouville problem with symmetric potential and nonseparated boundary conditions is unique. The unique reconstruction of Sturm-Liouville problems is accomplished by these theorems using a spectrum and two (or three) eigenvalues. The theorems apply the traditional Sturm-Liouville results of G. Borg and N. Levinson to the case of problems with general boundary conditions. With symmetric potential and general boundary conditions, schemes for the original reconstruction of Sturm-Liouville problems are provided.

Keywords: boundary conditions, the inverse eigenvalue problem, general inverse Sturm-Liouville, problem symmetric potential

1. Introduction

Let L stands for the Sturm-Liouville problem

$$Ly = -y''(x) + p(x)y(x) = \lambda y(x) \quad (1)$$

$$U_i(y) = a_{i1}y(0) + a_{i2}y'(0) + a_{i3}y(1) + a_{i4}y(1) = 0, \quad (2)$$

Where $p(x) \in L_1(0,1)$ is a real function so that $p(x) = p(1-x)$ the virtually everywhere (a.e.) a_{ij} with $i = 1,2$ and $j = 1,2,3,4$ are complex constants.

The inverse Sturm-Liouville problem for L in the case of separated boundary conditions and the boundary value problem for second-order differential equations $(a_{13} = a_{14} = a_{22} = a_{22} = 0)$ have undergone thorough research (see [1, 2, 3, 4, 6,9, 11, 13, 14, 15, 16]). Researchers V.A.Sadovnichii, V.A.Yurko, V.A.Marchenko, O.A.Plaksina, I.M.Guseinov, and I.M.Nabiev investigated the inverse Sturm-Liouville problem with unknown coefficients in nonseparated boundary conditions (see [4, 5, 8, 9, 14, 16,]).

For the inverse reconstruction problem, L in all coefficients of which a_{ij} with $i=1,2$ and $j=1,2,3,4$ are unknown, no uniqueness theorems have been proved. Special cases of problem L with boundary conditions

$$V_1(y)=a_{11}y(0)+y'(0)+a_{13}y(1)=0, \quad (3)$$

$$V_2(y)=a_{21}y(0)+y'(1)+a_{23}y(1)=0, \quad (4)$$

and

$$W_1(y)=y(0)+\beta\, y(1)=0, \quad (5)$$

$$W_2(y)=\overline{\beta}y'(0)+y'(1)+\alpha\, y(1)=0, \quad (6)$$

previously researched. The following sorts of generic self-adjoin nonseparated border conditions (2) can be reduced to: the boundary conditions (3),(4), where a_{11} and a_{23} if any numbers are real, $a_{13}\neq 0$ is any complex number, $a_{21}=-a_{13}$, also the boundary conditions (5), (6), where $\beta\neq 0$ where is any complex number and is any real integer.

In addition to the problem's spectrum, the spectra of two other boundary value problems, a specific set of signs, a specific real number were employed in order to uniquely recreate these boundary value problems with asymmetric potential (see, e.g., [4,6]). Using symmetric potential and general boundary conditions (2), which may not be self-adjoint, we demonstrate a theorem in this study about the unique reconstruction of problem L. The only spectrum data used are the eigenvalues of three spectral problems.

2. Objectives of this research

The purpose of this research paper is discussion for the problems of homogeneous linear differential equations of the second order by Sturm-Liouville boundary conditions of the general inverse with Symmetric Potential problem.

3. Methodology

Information has been collected in the form of libraries, websites, domestic and foreign scientific articles, undergraduate and doctoral research dissertations.

4. Literature review

Differential equations have been developed for nearly 300 years, and the relationship between evolutions are functions and derivatives of functions, so its history naturally dates back to the discovery of the derivative by the English scientist Isaac Newton (1642-1772) and Gottfried Wilhelm Leibniz (Germany (1646-1716)) began. Newton worked on differential equations, including first-order differential equations, into forms. Jacob proposed the Bernoulli differential equation in (1674), but failed to prove it until Euler proved it in (1705).

In the linear differential equations of the boundary problem, Sturm-Lowville first worked, the Sturm-Lowville theory in mathematics and its applications, the classical Sturm-Lowville theory, named after Jacques François Sturm (1803-1855) and Joseph

Lowville (1809-1882), the theory of linear differential equations is the second real order of form. In 1969, the Russian scientist Nymark wrote in his book Linear Differential Operators about the Green function to solve differential equations with boundary problem conditions.

The inverse Sturm-Liouville problem for L in the case of separated boundary conditions and the boundary value problem for second-order differential equations $(a_{13} = a_{14} = a_{22} = a_{22} = 0)$ have undergone thorough research research (see [1, 2, 3, 4, 6,9, 11, 13, 14, 15, 16].)

Researchers V.A. Sadovnichii, V.A. Yurko, V.A. Marchenko, O.A. Plaksina, I.M. Guseinov, and I.M. Nabiev investigated the inverse Sturm-Liouville problem with unknown coefficients in nonseparated boundary conditions (see [4, 5, 8, 9, 14, 16,]).

5. Borg's Uniqueness Theorems Generalization

For the inverse Sturm-Liouville problem's one and two with $q(x) \in L_1(0,1)$ solution in 1946, Borg established a number of uniqueness theorems [6, p. 69]. Two of them mentioned the spectrum issues listed below.

1. $ly = -y''(x) + p(x)y(x) = \lambda y(x)$, $y(0) = 0$, $y(1) = 0$, $p(x) = p(1-x)$ *a.e.*
2. $ly = -y''(x) + p(x)y(x) = \lambda y(x)$, $y'(0) = 0$, $y'(1) = 0$, $p(x) = p(1-x)$ *a.e.*

Borg proved the following theorems (in Borg's notation) for these problems [6, p. 69].

Theorem 1. The spectrum of Problem (1) is the sole determinant of the function $p(x)$ if $p(x) = p(x-1)\, a.e.$

Theorem 2. If $p(x) = p(x-1)\, a.e.$, then the spectrum of Problem (2) determines the function $p(x)$ in (1) uniquely.

To the problem of general boundary conditions, we generalize these theorems in this work (2).

In the terms that follow, we'll refer to a problem of type L as e $\tilde{L}$. This problem has a separated set of equation coefficients and boundary form parameters. We assume that if a symbol designates an object from Problem L, then the identical symbol with the tilde $\tilde{L}$ designates the same object from Problem $\tilde{L}$ throughout the work.

Let M denote the matrix coefficients a_{ik} of the boundary conditions' (2), i. e. ,

$$M = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{pmatrix}$$

and let A_{ij} be its minors composed of i th and j th columns

$$A_{ij} = \begin{vmatrix} a_{1i} & a_{1j} \\ a_{2i} & a_{2j} \end{vmatrix}, \quad i, j = 1,2,3,4.$$

Boldface letters are used to denote vectors. Transposition is denoted by the symbol T. Rows with this superscript serve as column vector representations. We use the notation rank M to represent the rank of the matrix M.

Problems are also present in space of L, (1) and (2) we consider the following Problems

$$ly = -y''(x) + p(x)y(x) = \lambda y(x),$$
$$U_{1,1}(y) = y(0) - \kappa(\lambda)y'(0) = 0,$$
$$U_{2,1}(y) = y(1) = 0,$$

and

$$ly = -y''(x) + p(x)y(x) = \lambda y(x),$$
$$U_{1,1}(y) = y'(0) - \kappa(\lambda)y(0) = 0,$$
$$U_{2,1}(y) = y'(1) = 0,$$

In Problems $\kappa(\lambda)$ is a polynomial of the form:

$$\kappa(\lambda) = A_{12} + (1 - A_{13})\lambda + (A_{14} - A_{32})\lambda^2 + A_{42}\lambda^3 + A_{34}\lambda^4.$$

Theorem 3. If Problems L and $\tilde{L}$ have a nonempty discrete spectrum; the spectra of problems L and $\tilde{L}$, (1) and $(\tilde{1})$, L_1 and $\tilde{L}_1$ coincide with algebraic multiplicities taken into account, and rank M=2 , then these boundary value problems themselves coincide, i.e., $p(x) = \tilde{p}(x)$ a.e. and the matrices $M = (a_{i\,j})_{2\times 4}$ and $\tilde{M} = (\tilde{a}_{i\,j})_{2\times 4}$

Proof. When we refer to Problem (1) Borg's uniqueness theory P1 for the inverse Sturm-Liouville problem with symmetric potential [6, p. 69], we observe that

$$p(x) = \tilde{p}(x) \ a.e. \quad (7)$$

Let us show that, for the vectors $N = (A_{12}, A_{13}, A_{14}, A_{32}, A_{42}, A_{34})^T$ and $\tilde{N} = (\tilde{A}_{12}, \tilde{A}_{13}, \tilde{A}_{14}, \tilde{A}_{32}, \tilde{A}_{42}, \tilde{A}_{34})^T$ composed of the minors of the matrices $(a_{ij})_{2x4}$ and $(\tilde{a}_{ij})_{2x4}$ respectively, we have:

$$N = \tilde{N} \quad (8)$$

Let $y_1(x,\lambda)$ and $y_2(x,\lambda)$ be linearly independent solutions to Equation (1) that meet the conditions

$$y_1(0,\lambda) = 1, \qquad y_1'(0,\lambda) = 0, \qquad y_2(0,\lambda) = 0, \qquad y_2'(0,\lambda) = 1. \quad (9)$$

The roots of the entire function are the Problem L eigenvalues. ([17, pp. 33–36], [19, p. 29].

$$\Delta(\lambda) = A_{12} + A_{34} + A_{32}y_1(\pi,\lambda) + A_{42}y_1'(\pi,\lambda) + A_{13}y_2(\pi,\lambda) + A_{14}y_2'(\pi,\lambda), \quad (10)$$

and the roots of the entire function are Problems L_1 eigenvalues.

$$\Delta_1(\lambda) = -\phi(\lambda)y_1(\pi,\lambda) + y_2(\pi,\lambda)$$

If $\Delta(\lambda)\neq 0$ (i.e., the spectrum of the boundary value problem is discrete), Hadamard's Theorem states that the function $\Delta(\lambda)$ can be reconstructed from its zeros up to a factor $C\neq 0$. Therefore, the functions $\Delta(\lambda)$ and $\tilde{\Delta}(\lambda)$ are related by the identity

$$\frac{\Delta(\lambda)}{\tilde{\Delta}(\lambda)}=C \qquad (11)$$

Where C is a nonzero constant.

If $\Delta(\lambda)\equiv 0$ (i.e., each λ is an eigenvalue of Problem L), subsequently, the condition that the eigenvalues of Problems L and $\tilde{L}$ coincide also implies (11) whence $\tilde{\Delta}(\lambda)\equiv 0$. Similarly, we have:

$$\frac{\Delta_1(\lambda)}{\tilde{\Delta}_1(\lambda)}=C_1$$

Where C_1 is a nonzero constant.

The following asymptotic relations hold:

$$y_1(x,\lambda)=\cos\sqrt{\lambda}x+\frac{1}{\sqrt{\lambda}}f(x)\sin\sqrt{\lambda}x+\mathrm{O}(\frac{1}{\lambda}),$$

$$y_1'(x,\lambda)=-\sqrt{\lambda}\sin\sqrt{\lambda}x+f(x)\cos\sqrt{\lambda}x+\mathrm{O}(\frac{1}{\sqrt{\lambda}}),$$

$$y_2(x,\lambda)=\frac{1}{\sqrt{\lambda}}\sin\sqrt{\lambda}x-\frac{1}{\lambda}f(x)\cos\sqrt{\lambda}x+\mathrm{O}(\frac{1}{\lambda\sqrt{\lambda}}),$$

$$y_2'(x,\lambda)=\cos\sqrt{\lambda}x+\frac{1}{\sqrt{\lambda}}f(x)\cos\sqrt{\lambda}x+\mathrm{O}(\frac{1}{\lambda}),$$

Where $f(x)=\frac{1}{2}\int_0^x q(t)dt$ and for sufficiently large $\lambda\in IR$.

These connections imply that the functions $y_1(\pi,\lambda)$ and $y_2(\pi,\lambda)$ in the decomposition of the function $\Delta_1(\lambda)$ are linearly independent. Therefore,

$$A_{12}=\tilde{A}_{12},\qquad A_{13}=\tilde{A}_{13},\qquad A_{42}=\tilde{A}_{42},\qquad A_{34}=\tilde{A}_{34},\qquad (12)$$

The functions $y_1(\pi,\lambda)=y_2'(\pi,\lambda)$, and 1 in the decomposition of $\Delta(\lambda)$ are linearly independent as well (the relation $y_1(\pi,\lambda)=y_2'(\pi,\lambda)$ holds if and only if $p(x)=p(x-\pi)$.

These observations, together with (10) and (11), implies

$$\frac{A_{12}+A_{34}}{\tilde{A}_{12}+\tilde{A}_{34}}=C$$

$$\frac{A_{14}+A_{32}}{\tilde{A}_{14}+\tilde{A}_{32}}=C \qquad (13a)$$

$$\frac{A_{42}}{\tilde{A}_{42}}=C$$

$$\frac{A_{13}}{\tilde{A}_{13}}=C \qquad (13b)$$

At least one of the numbers $A_{12}+A_{34}$, $A_{32}+A_{14}$, A_{42} *and* A_{13} is different from zero. Alternatively, we would have $\Delta(\lambda)\equiv 0$ contradiction to the theorem's presumption that Problems L and $\widetilde{L}$ have discrete spectrum. This finding, along with (12), (13a) and (13b) suggests

If $C=1$ then the result we have:

$$A_{12}=\widetilde{A}_{12},\quad A_{13}=\widetilde{A}_{13},\quad A_{14}=\widetilde{A}_{14},\quad A_{32}=\widetilde{A}_{32},\quad A_{42}=\widetilde{A}_{42},\quad A_{34}=\widetilde{A}_{34},$$

Whence we obtain (8).

From (8) it follows that the matrices $(a_{ij})_{2\times 4}$ and $(\widetilde{a}_{ij})_{2\times 4}$ coincide up to a linear transformation of the rows (see [4, p. 32]). Combining this with (7), it becomes clear that the boundary value problems L and $\widetilde{L}$ are related.

Under certain conditions, the following theorem (stronger than Theorem 1) holds true.

6. Levinson's Uniqueness Theorem Generalizations

In 1949, Levinson considered the following Sturm-Liouville problem L_0 with symmetric potential [12]

Problem L_0:

$$Ly=-y''(x)+p(x)y(x)=\lambda\, y(x),$$
$$y'(0)-h\,y(0)=0,\qquad y'(\pi)+h\,y(\pi)=0,\qquad h\in IR.$$

Levinson provided the following theorem for this problem.

Theorem. If $p(x)=p(x-\pi),$ The spectrum of Problem L_0 then determined uniquely the function $p(x)$ and the number of h.

This section expands on this theorem to encompass nonseparated boundary conditions.

Consider the following spectral problem.

Problem 3:

$$Ly=-y''(x)+p(x)y(x)=\lambda\, y(x),$$
$$U_{1,1}(y)=a_{11}y(0)-a_{21}y(\pi)+y'(0)=0,$$
$$U_{2,1}(y)=a_{21}y(0)+y'(\pi)+a_{23}y(\pi)=0,\qquad a_{11},a_{21},a_{23}\in IR.$$

Problem 3 boundary conditions are the boundary conditions (3) and (4), where a_{11}, a_{13}, a_{23}, and a_{21} are any real numbers and $a_{21}=-a_{13}$. in (33, YURKO) showed that problem 3 can be uniquely recreated using two spectra and a series of indicators, namely from the spectrum of Problem 3, the spectrum $\{Z_n\}$ of the problem for equation (2) and boundary conditions $y'(0)+a_{11}y(0)=y(\pi)=0,$ and the sequence of signs $\omega_n=sign\left(\left|\psi'(\pi,Z_n)\right|-\left|a_{21}\right|\right),$ where $\psi(x,\lambda)$ is the solution of equation (2) under the boundary conditions $\psi(0,\lambda)=0$, $\psi'(0,\lambda)=-a_{11}$.

In what follows, we show that if the potential of Problem 3 is symmetric, then Problem 3 can be reconstructed from two spectra (a sequence of signs is not needed in this case).

Let 4 denote the following spectral problem.

Problem 4.

$$Ly = -y''(x) + p(x)y(x) = \lambda\, y(x),$$
$$a_{11}\, y(0) + y'(0) = 0, \qquad -a_{11}\, y(\pi) + y'(\pi) = 0.$$

Theorem 4. If $p(x) = p(\pi - x),\ \tilde{p}(x) = \tilde{p}(\pi - x)$ and the spectral of Problems 3 and $\tilde{3}$, 4 and $\tilde{4}$ coincide with algebraic multiplicities taken into account, then these boundary value problems themselves coincide, i.e., $p(x) = \tilde{p}(x),\ \ a_{11} = \tilde{a}_{11},\ \ a_{21} = \tilde{a}_{21}$, and $a_{23} = \tilde{a}_{23}$.

Proof. Using problem 4 as an example, we can see that, for the inverse Sturm-Liouville problem with symmetric potential, Levinson's uniqueness theorem [12] is true.

$$p(x) = \tilde{p}(x),\ \ a_{11} = \tilde{a}_{11}. \quad (14)$$

The relations still need to be proved in order to verify the theorem $a_{21} = \tilde{a}_{21}$ and $a_{23} = \tilde{a}_{23}$. Problem 3 eigenvalues are the function's full roots.

$$\Delta_3(\lambda) = -a_{21} - a_{23} y_1(\pi,\lambda) - y_1'(\pi,\lambda) + (a_{11}a_{23} + a_{21}a_{21})\, y_2(\pi,\lambda) + a_{11}\, y_2'(\pi,\lambda) \quad (15)$$

According to Hadamard's theorem, the function $\Delta_3(\lambda)$ (which is entire of order 1/2) can be reconstructed from its zeros up to a multiplier $C \neq 0$. Therefore, the functions $\Delta_3(\lambda)$ and $\tilde{\Delta}_3(\lambda)$ are related by the identity

$$\frac{\Delta_3(\lambda)}{\tilde{\Delta}_3(\lambda)} = C_3 \quad (16)$$

where C is a nonzero constant. The asymptotic relations show that the functions $y_1(\pi,\lambda) = y_2'(\pi,\lambda),\ \ y_1'(\pi,\lambda),\ \ y_2(\pi,\lambda)$ and 1 are linearly independent.

Therefore, if $C = 1$ then the result we have:

$$a_{21} = \tilde{a}_{21},\ \ a_{23} = \tilde{a}_{23}.$$

under certain conditions, stronger results than Theorem 4 hold true.

Let's say that the value a_{11} and the function $p(x)$ are rebuilt. Then, given conditions (9), the linearly independent solutions $y_1(x)$ and $y_2(x)$ of equation (1) are known. As a result, we can establish the following conditions.

Condition 1. Numbers λ_1 and λ_2 satisfy equations

$$y_2(\pi,\lambda_1) = y_2(\pi,\lambda_2) = 0.$$

and inequalities

$$y_1(\pi,\lambda_1) \neq y_1(\pi,\lambda_2) \neq 0.$$

Condition 1. Numbers λ_1, λ_2 and λ_3 satisfy equations

$$\begin{vmatrix} 1 & y_1(\pi,\lambda_1)-a_{11}y_2(\pi,\lambda_1) & y_2(\pi,\lambda_1) \\ 1 & y_1(\pi,\lambda_2)-a_{11}y_2(\pi,\lambda_2) & y_2(\pi,\lambda_2) \\ 1 & y_1(\pi,\lambda_3)-a_{11}y_2(\pi,\lambda_3) & y_2(\pi,\lambda_3) \end{vmatrix} \neq 0.$$

7 conflict

In this research, it has been determined that a Storm-Lewell self-adjoint inverse problem with symmetric potential and non-separated boundary conditions is unique. In addition, the unique reconstruction of Sturm-Liouville problems is performed by these theorems using one-spectrum and two-spectrum (or three-spectrum) eigenvalues. A unique reconstruction of Sturm-Liouville problems is performed by these theorems using a spectrum and two (or three) eigenvalues.

8. Conclusion

In this research, it has been determined that the second-order differential equations with boundary problem conditions and eigenvalue conditions on a Storm-Liouville self-adjoint inverse problem with symmetric potential and non-separated boundary conditions become unique.

In addition, a unique reconstruction of Sturm-Liouville problems using one spectrum and two spectra (or three spectra) of eigenvalues is performed. A unique reconstruction of Sturm-Liouville problems is performed by these theorems using a spectrum and two (or three) eigenvalues.

References

1. A.R. Aliev .(2006). On the generalized solution of the boundary-value problem for the operator differential equations of the second order with variable coefficients. Journal of Mathematical Physics, Analysis, Geometry. 2(1).

2. A.R. Aliev. (2006) On the solvability of a class of operator differential equations of the second order on the real axis. Journal of Mathematical Physics, Analysis, Geometry.2(4).

3. A.M. Akhtyamov.(2009). Identification Theory of Boundary Value Problems and Its Applications. Moscow Russia.

4. Aimal Rasa Ghulam hazrat.(2021). Asymptotic Formulas for Weight Numbers of the Boundary Problem differential operator on a Star-shaped Graph. Turkish Journal of Computer and Mathematics Education.12(13). Trabzon, Turkey.

5. Aimal Rasa Ghulam hazrat. and Gaukhar Auzerkhan .(2021). Inception of Green function for the third order liner differential equation that inconsistent with the boundary conditions. Journal of Mathematics, Mechanics and Computer Science. V(110), N 2.Almaty, Kazakhstan.

6. Aimal Rasa Ghulam hazrat (2022). Formulas for Surface Weighted Numbers on Graph. International Journal of Trend in Scientific Research and Development. V (6). Hindia.

7. B.M. Levitan .(1984). Inverse Sturm-Liouville problems. Moscow, Russian.

8. B.M. Levitan. (1964). Determination of a Sturm-Liouville differential equation in terms of two Spectra. Izv. Akad. Nauk SSSR Ser. Mat., 28. Russia.

9. B.M. Levitan, M.G. Gasymov. (1964). Determination of a Differential Equation by Two of Its Spectra. Uspekhi Mat. Nauk, 19(2). Russia.

10.B.M. Levitan. (1984) Inverse Sturm-Liouville problems. Nauka, Moscow, Russia.

11. Б.Е Кангужин, Даирбаева Г.,Мадибайулы Ж. .(2019). Journal of Mathematics, Mechanics and Computer Science. V (103), №3. Almaty, Kazakhstan.

12. Kanguzhin, B.; Aimal Rasa, G.H.; Kaiyrbek, Z.(2021) Identification of the domain of the sturm–liouville operator on a star graph. Symmetry Journal, V (13) № 1210.

13. E.S. Panakhov, H. Koyunbakan. (2007). Half inverse problem for diffusion operators on the finite interval. Journal of Mathematical Analysis and Applications .(326), 10241030.

14. G. Borg. (1946). Inversion of the Sturm-Liouville eigenvalue problem. Acta Math.

15. I.M. Guseinov, I.M. Nabiev .(2007). An inverse spectral problem for pencils of differential. Mat. Sb, 198(11). Russian.

16. M.A. Naimark. (1969). Linear Differential Operators. 2nd ed., Nauka. Moscow, Russian.

17. O.A. Plaksina. (1986). Inverse problems of spectral analysis for the SturmLiouville opera- tors with nonseparated boundary conditions. Mat. Sb.131(1).

18. O.A. Plaksina. (1988). Inverse problems of spectral analysis for the Sturm-Liouville operators with nonseparated boundary conditions. Mat. Sb., 136(1). Russian.

19. V.A. Marchenko .(1977). SturmLiouville operators and their applications. Naukova Dumka, Kiev, Ukraine.

20. V.A. Sadovnichii. (1972). Uniqueness of the Solution of the Inverse Problem for Second Order Differential Equation with Nonseparated Boundary Conditions. Regularized Sums of Eigenvalues, Factorization of the Characteristic Determinant. Dokl. Akad. Nauk SSSR, 206(2).

21. V.A. Sadovnichii, Ya.T. Sultanaev,A.M. Akhtyamov. (1999). Analogues of Borgs Uniqueness Theorem in the Case of Nonseparated Boundary Conditions. Doklady Akad. Nauk, 367(6). Russian.