Application of derivatives in economic problems

Nodir Giyosalievich Khidirov
Tashkent Institute of Finance

Abctract: This article shows the use of mathematical methods in solving economic problems, such as the application of derivatives to the basic laws of the theory of production and consumption, as well as supply and demand.

Keywords: factors of production, demand, supply, fixed costs, variable costs, benefits

An adept economist in contemporary times is expected to possess proficiency in quantitative analytical methods - an expectation evident early in the study of economic theory. This proficiency extends beyond traditional mathematical courses such as mathematical analysis, linear algebra, and probability theory, encompassing knowledge essential for practical applications in economics and economic research. Proficiency in mathematical and economic statistics, game theory, econometrics, and related disciplines is paramount.

Mathematics, in this context, serves not merely as a tool for quantitative calculations but as a method facilitating precise research. It acts as a vehicle for the meticulous formulation of economic concepts and problems, offering a foundation for clear and exact exploration in the field of economics.

It has long been observed that "only differential calculus provides the natural sciences with the ability to mathematically represent not just states, but also processes: motion". Consequently, the objective of my work is to discern the economic significance of the derivative, elucidate the novel avenues that differential calculus unveils for economic research, and delve into the applications of the derivative in addressing diverse problem sets within economic theory [1].

1. The derivative stands out as a paramount tool in economic analysis, enabling a deeper exploration of the geometric and mathematical nuances of economic concepts. Moreover, it facilitates the articulation of various economic laws through mathematical formulas.

2. Leveraging the power of the derivative significantly broadens the spectrum of functions under consideration when tackling problem-solving in economic contexts.

3. The economic interpretation of the derivative is clear: it serves as the measure of the rate of change in an economic process over time or in relation to another factor under examination.

4. The derivative finds its most pertinent application in marginal analysis, precisely in the examination of marginal values - be it marginal costs, marginal

revenue, or the marginal productivity of factors such as labor or other elements in the production process.

5. The derivative holds broad applicability in economic theory. Numerous fundamental laws in the realms of production and consumption, supply and demand, are revealed as direct outcomes of mathematical theorems—illustrating, for instance, the economic interpretation of Fermat's theorem and the significance of function convexity.

6. Proficiency in the derivative empowers individuals to navigate and resolve a myriad of problems within economic theory.

Using the derivative to solve problems in economic theory

Task 1: The demand function is expressed as $Q_D = 100 - 20p$ with total fixed costs (TFC) set at 50 monetary units and total variable costs (TVC) for producing a unit of output amounting to 2 monetary units. Determine the output volume that maximizes the monopolist's profit.

Solution: Profit is derived from subtracting costs from revenue.

$$P = TR - TC$$

Where:

$TR = p * Q;$

$TC = TFC + TVC.$

To determine the unit price:

$$20p = 100 - q \implies p = 5 - Q/20$$

Then

$$P = \left(5 - \frac{Q}{20}\right)Q - (5 + 2Q) = -Q^2 + 60Q - 1000 \longrightarrow max$$

Find the derivative: $P'(Q) = -2Q + 60$

Equate the derivative to zero:$-2Q + 60 = 0 \implies Q = 30(Q)$

When traversing through the point Q = 30, the function $P(Q)$ undergoes a change in sign from positive to negative. Consequently, this point represents the maximum point, and at this juncture, the profit function attains its highest value. Thus, the output level that maximizes profit is 30 units.

Task 2: The demand for the enterprise's products is represented by the formula $Q_D = 200 - 4p$, and the supply volume is given by $Q_S = 6p - 100$. The variable costs per unit of production are *TVC=25*. What should the unit price *(p)* be for the profit *(Π)* to be maximized?

Solution: At the point of consumer equilibrium where $Q_S = Q_D$,

i.e.

$$6p_0 - 100 = 200 - 4p_0$$

At the equilibrium point, where $p_0 = 30$ (monetary units), the corresponding equilibrium volume of production $\implies Q_0 = 80$ units.

Let us graphically depict the supply and demand curves, as well as the consumer equilibrium point located at their intersection.

Let's consider three possible options:

1) $p > p_0 \Longrightarrow Q = Q_D$, i.e. $P = Q_0 p - Q_D TVC = Q_D(p - TVC)$,

Substitute the values and get:

$$P = (200 - 4p) * (p - 25) = -4p^2 + 300p - 5000$$

2) $p = p_0, \Longrightarrow Q = Q_D = Q_S \Longrightarrow Q_{sale} = Q_0 = 80\ (unit), \Longrightarrow\ P_2 = 80 * (30 - 25) = 400\ (monetary\ unit)$

3) $p < p_0 : \Longrightarrow Q = Q_S$, i. e. $P = Q_S p - Q_S TVC = Q_S(p - TVC)$,

Substitute the values:

$$P = (6p - 100) * (p - 25) = 6p^2 - 250p - 2500$$

Further, cases (1) and (3) can be solved analytically by substituting different price values from the interval of its values or in some other way, but it is much easier to identify profit extremes through the derivative:

1) $P = -4p^2 + 300p - 5000$

$P' = -8p + 300;$

$$-8p + 300 = 0 \ \Longrightarrow p = \frac{75}{2} = 37{,}5\ (monetary\ unit)$$

As the result, $Q = Q_D = 200 - 4 * 37{,}5 = 200 - 150 = 50\ (mon.unit)$

$$P_1 = -4p^2 + 300p - 5000 = -4(37{,}5)^2 + 300 + 37{,}5 - 5000 \\ = 625\ (mon.unit)$$

2) In the second case, the profit has already been found: $\Pi_2 = 400\ (mon.unit)$

3) $P = 6p^2 - 250p - 2500$

$P' = 12p - 250$

$$12p - 250 = 0 \Longrightarrow p = \frac{125}{6} = 20\frac{5}{6}\ (mon.unit)$$

As the result, $Q = Q_S = 6 * 20\frac{5}{6} - 100 = 125 - 100 = 25\ (\ unit)$,

$$P_3 = 6p^2 - 250p + 2500 = 6 * \left(20\frac{5}{6}\right)^2 - 250 * 20\frac{5}{6} + 2500 \\ = -104\frac{1}{6}\ (mon.unit)$$

We can deduce that profit is maximized in the first scenario; hence, the optimal unit price should be 37.5 monetary units.

Task 3: Determine the monopolist's maximum revenue if the demand, up to the point of intersection with the axes, is characterized by the linear function й $Q = b - ap$, where p is the price of the monopolist's goods, *a* and *b* are the coefficients of the demand function.

Solution: Revenue $TR = Qp = p(b - ap)$ will reach its maximum when the price derivative is equal to zero:

$$TR' = (p(b - ap))' = 0$$

$$TR' = p' * (b - ap) + (b - ap)' * p = b - ap - ap = b - 2ap \Rightarrow p = \frac{b}{2a}$$

$$\Rightarrow Q = b - ap = b - a * \frac{b}{2a} = \frac{b}{2}$$

In this case, the maximum revenue will be

$$TR = Qp = \frac{b}{2} * \frac{b}{2a} = \frac{b^2}{4a}$$

References

1. Kasman L.M, Kasman A., Westwater C., Dolan J., Schmidt M.G., Norris J.S. Overcoming the phage replication threshold: a mathematical model with implications for phage therapy. Journal of Virology. 2002 Jun;76(11):5557-64. doi: 10.1128/jvi.76.11.5557-5564.2002. PMID: 11991984; PMCID: PMC137030.

2. Розен В.В. Математические модели принятия решений в экономике. -М.: Высш.шк., 2002.

3. Малихын В.Л. Математика в экономике. -М.: "ИНФРА-М", 2001.

4. Sotvoldiyev, A. I., & Xidirov, N. G. (2022). Dinamik modellarni iqtisodiyotda qo'llanilishi. Science and Education, 3(3), 1128-1137.

5. Khidirov, N. G. (2023). EVALUATION OF EFFICIENCY OF INVESTMENT ACTIVITY IN INDUSTRIAL ENTERPRISES. World Economics and Finance Bulletin, 22, 29-32.

6. Sotvoldiyev, A. I. (2023). Mathematics of economic processes nature and methods of modeling. Science and Education, 4(3), 829-835.

7. Abchuk V.A. Economic and mathematical method: Elementary mathematics and logic. Operative method issledovaniya. SPb.: Soyuz, 1999.

8. Фозилчаев, Ш. Қ., & Хидиров, Н. Ғ. (2017). Инвестиция ва лизинг асослари. Ўқув қўлланма. Т.: Молия, 8, 46-48.

9. Хидиров, Н. (2020). Анализ источников финансирования инвестиционной деятельности текстильных предприятий. Экономика И Образование, 1(3), 71-77.