Tenglama va tenglamalar sistemasini rezultant usulida yechish

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Annotatsiya: Ushbu maqolada ko’phadlar umumiy ildizi bor yo’qligi, tenglama va tenglamalar sistemasini rezultant usulida yechish, ko’phad karrali ildizlari bor yoki yo’qligini bir necha usullari ko’rsatilgan.

Kalit so’zlar: ko’phad, rezultant, tenglamalar, tenglamalar sistemasi, karrali ildiz

Solving equations and systems of equations by the resultant method

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Abstract: In this article, several methods are shown to determine whether polynomials have common roots, to solve equations and systems of equations using the resultant method, to determine whether polynomials have multiple roots.

Keywords: polynomial, resultant, equations, system of equations, multiple root

P maydonda ko’phadlar rezultantini topishning:

I usuli:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + ...+ a_1 x + a_0$$
$$g(x) = b_m x^m + b_{m-1} x^{m-1} + ...+ b_1 x + b_0$$

Ko’phadlar berilgan bo’lsin va $a_n b_m \neq 0$ va $\alpha_1, \alpha_2, ...\alpha_n$ lar $f(x)$ ning ildizlari, $\beta_1, \beta_2, ...\beta_m$ lar esa $g(x)$ ning ildizlari bo’lsin.

$f(x)$ *va* $g(x)$ *ko’phadlarning rezultanti*

$$R(f, g) = a_n^m g(\alpha_1) * g(\alpha_2) * ....g(\alpha_n) \quad (1)$$

dan iborat.

Agar $\beta_1, \beta_2, ...\beta_m$ lar esa $g(x)$ ning ildizlari bo’lsa, u holda

$$R(f,g)=a_n^m b_m^n \prod_{\substack{1\le i\le n\\ 1\le j\le m}} (\alpha_i-\beta_j)$$

$$R(g,f)=(-1)^{mn}R(f,g)$$

o'rinli bo'ladi.

II usuli:

$f(x)$ *va* $g(x)$ ko'phadlarning rezultantini topishning yana bir usuli

Silvestr formulasi yordamida rezultant quyidagicha topiladi:

$$R(f,g)=\begin{vmatrix} a_n & a_{n-1} & {}_1.. & 0 & ... & 0 \\ 0 & a_n & .. & a_0 & ... & 0 \\ ... & ... & ... & ... & ... & ... \\ b_m & b_{m-1} & .. & 0 & .. & 0 \\ .. & .. & .. & .. & .. & .. \\ 0 & .. & b_m & b_{m-1} & .. & 0 \end{vmatrix} \quad (2)$$

Teorema: *f(x)* va *g(x)* ko'phadlar umumiy ildizga ega bo'lishi uchun *R(f,g)=0* bo'lishi zarur va yetarli.

f(x) ko'phadning diskriminanti:

$$D(f)=(-1)^{\frac{n(n-1)}{2}}a_n^{-1}R(f,f^{-1})$$

yoki

$$D(f)=a_n^{2n-2}\prod_{1\le i\le j\le n}(a_i-a_j)^2$$

1-Misol. $f(x)=x^2-3x+6$ va $g(x)=x^3+x^2-x-1$ ko'phadlar uchun rezultant toping.

Yechish: $f(x)$ va $g(x)$ ko'phadlarning rezultantini topish uchun yuqoridagi I usuldan foydalanamiz.

$f(x)$ ko'phadning ildizlar kompleks sonlardan iborat, hisoblashga biroz noqulaylik tug'diradi shuning uchun $g(x)$ ko'phadning ildizlarini topamiz:

$$g(x)=x^3+x^2-x-1$$

$$g(x)=x^2(x+1)-(x+1)$$

$$g(x)=(x+1)(x^2-1)$$

$$g(x)=(x+1)^2(x-1)$$

$$x=\pm 1.$$

$R(f,g)=(-1)^{3*2}R(g,f)=R(g,f)$ bo'lganligi uchun va (1) ga asosan

$R(f,g)=f(-1)*f(-1)*f(1)=10*10*4=400$.

$f(x)$ va $g(x)$ ko'phadlarning rezultanti 400 ga teng.

Javob: $R(f,g)=400$

2-Misol. $f(x)=2x^3-3x^2-x+2$ *va* $g(x)=x^4-2x^2-3x+4$ ko'phadlarni rezultantini toping.

Yechish: $f(x)$ va $g(x)$ ko'phadlarning ildizlarini topish biroz muammo bo'lganligi sababli , rezultantini topish uchun Silvestr formulasidan foydalanamiz;

$$R(f,g)=\begin{vmatrix} 2 & -3 & -1 & 2 & 0 & 0 & 0 \\ 0 & 2 & -3 & -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & -3 & -1 & 0 & 0 \\ 0 & 0 & 0 & 2 & -3 & -1 & 2 \\ 1 & 0 & -2 & -3 & 4 & 0 & 0 \\ 0 & 1 & 0 & -2 & -3 & 4 & 0 \\ 0 & 0 & 1 & 0 & -2 & -3 & 4 \end{vmatrix}=.....=0$$

$R(f,g)=0$ ekanligidan $f(x)$ va $g(x)$ ko'phadlarning umumiy ildiziga ega, ya'ni EKUBi mavjud.

Javob: $R(f,g)=0$.

3-Misol. $\begin{cases}\sqrt{x}+\sqrt{y}=9\\ \sqrt[3]{x}+\sqrt[3]{y}=5\end{cases}$ R maydonda tenglamalar sistemasini yeching

Yechish: $\begin{cases}\sqrt{x}+\sqrt{y}=9\\ \sqrt[3]{x}+\sqrt[3]{y}=5\end{cases}$ bu tenglamalar sistemasini yechish uchun birinchi navbatda belgilash kiritamiz:

$$\sqrt[6]{x}=a$$
$$\sqrt[6]{y}=b \quad (3)$$

Natijada:

$$\begin{cases}a^3+b^3=9\\ a^2+b^2=5\end{cases}$$

ga ega bo'ldik. Bu tenglamalar sistemasida har bir tenglamani alohida funksiya deb olsak, funksiyalar umumiy ildizga ega bo'lishi uchun rezultanti 0 ga tenglab, qiymatlarni topamiz

$$\begin{cases}a^3+b^3=9\\ a^2+b^2=5\end{cases}\Rightarrow\begin{vmatrix} 1 & 0 & b^2-5 & 0 & 0 \\ 0 & 1 & 0 & b^2-5 & 0 \\ 0 & 0 & 1 & 0 & b^2-5 \\ 1 & 0 & 0 & b^3-9 & 0 \\ 0 & 1 & 0 & 0 & b^3-9 \end{vmatrix}=0\Rightarrow b^9+b^6-15b^4-18b^3+75b^2-44=0$$

$$(b-1)(b^8+b^7+b^6+2b^5+2b^4-13b^3-31b^2+44b-44)=0$$

Bundan: $b=1$. Natijada, $a=2$ qiymatni topamiz. *a* va *b* qiymatlarni (3)ga qo'ysak:

$$\sqrt[6]{x}=2 \qquad x=64$$
$$\sqrt[6]{y}=1 \qquad y=1$$

ga ega bo'lamiz.

Javob: $x=64, \ y=1$

4-Misol.Tenglamalarni yeching: $x+\sqrt{17-x^2}+x\sqrt{17-x^2}=9$.

Yechish: 3-misoldagi kabi tenglamalar sistemasini yechishning Rezultant usulidan (Silvestr detirminantidan) foydalanamiz.Buning uchun quyidagicha belgilash kiritib olamiz:

$$\sqrt{17-x^2}=y$$

Natijada, 3-misoldagi tenglamalar sistemasiga ega bo'lamiz:

$$\begin{cases} x+y+xy=9 \\ 17-x^2=y^2 \end{cases} \Rightarrow \begin{cases} x(1+y)+y=9 \\ y^2+x^2=17 \end{cases}$$

$$\begin{vmatrix} 1+y & y-9 & 0 \\ 0 & 1+y & y-9 \\ 1 & 0 & y^2-17 \end{vmatrix}=0 \Rightarrow$$

$$\Rightarrow \begin{vmatrix} 1+y & y-9 & 0 \\ 0 & 1+y & y-9 \\ 1 & 0 & y^2-17 \end{vmatrix}\begin{matrix} 1+y & y-9 \\ 0 & 1+y \\ 1 & 0 \end{matrix}\Bigg| =$$

$$=(1+y)^2(y^2-17)+(y-9)^2-0=(1+2y+y^2)(y^2-17)+y^2-18y+81=$$
$$=y^2+2y^3+y^4-17-34y-17y^2+$$
$$+y^2-18y+81=y^4+2y^3-15y^2-52y+64=0$$
$$y^4+2y^3-15y^2-52y+64=0$$
$$(y-1)(y^3+3y^2-12y+64)=0$$
$$y=1.$$

Topilgan qiymatni belgilashga olib borib qo'ysak: $x=\pm4$ ga ega bo'lamiz.

Javob: $x=\pm4$

Misol-5

Quyidagi $f(x)=x^4-4x+3$ ko'phad karrali ildizga egami?

Yechish:

I-usuli.

$f(x)=x^4-4x+3$ ko'phadning hosilasini topami: $f'(x)=4x^3-4$.

Teorema: f(x) ko'phad karrali ildizga ega bo'lishi uchun *R(f,* $R(f(x), f'(x))=0$ bo'lishi zarur va yetarli.

Demak shu teoremaga asosan rezultant hisoblasak:

$$R(f, f') = \begin{vmatrix} 1 & 0 & 0 & -4 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 & -4 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 & -4 & 3 \\ 4 & 0 & 0 & -4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & -4 \end{vmatrix} = 0$$

Rezultant nolga teng bo'lganligi sababli karrali ildizga ega.

II usuli

$$f(x) = x^4 - 4x + 3$$

$$f(x) = x^4 - x^3 + x^3 - x + x - 4x + 3$$

$$f(x) = x^3(x-1) + x(x^2 - 1) - 3(x-1)$$

$$f(x) = (x-1)(x^3 + x^2 + x - 3) = (x-1)^2(x^2 + 2x + 3).$$

Demak, ko'phad karrali ildizga ega.

Ko'phadlar umumiy ildizlarini topishda rezultant usulidan foydalanish va buni tenglama va tenglamalar sistemasiga tadbiq qilish ildizlar topishning eng samarali usullaridan biri hisoblanadi.

Foydalanilgan adabiyotlar

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