Sonning butun va kasr qismi qatnashgan masalalarni yechish metodlari

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Annotatsiya: Sonning butun va kasr qismiga oid misollarni hamda ular qatnashgan tenglamalarni yechish usullari maktab matematika kursida, akademik litsey va kasb-hunar kollejlari darslik va o‘quv qo‘llanmalarida yetarli darajada ishlab chiqilmagan. Shuning uchun maqolada mavzu yuzasidan, ya’ni sonning butun va kasr qismiga hamda ular qatnashgan turli tenglamalarni yechish, ularni o‘quvchilarga tushuntirish metodikasini ishlab chiqish maqsad qilingan. Maqolada keltirib o‘tilgan metodik tavsiyalar umumta’lim maktablari, akademik litseylar o‘quvchilarida sonning butun va kasr qismi qatnashgan tenglamalarni ilmiy metodik jihatdan yechish usullarini takomillashtirishning muhim vositasi hisoblanadi. Shu bilan bir qatorda ushbu mavzudan o‘quv dasturlarini takomillashtirishda hamda matematika fanlari bo‘yicha o‘quv qo‘llanma va darsliklar ishlab chiqishda foydalanish mumkin.

Kalit so‘zlar: sonning butun va kasr qismi, ta’rif, tenglama, tengsizlik, masala hamda yechim

Methods of solving problems in which the integer and fractional part of a number are multiplicated

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Abstract: Methods of solving examples of whole and fractional parts of numbers and the equations involved in them are not sufficiently developed in the school mathematics course, academic lyceum and vocational college textbooks and training manuals. Therefore, the purpose of the article is to solve the whole and fractional part of the number and the various equations in which they are involved, and to develop a method of explaining them to the students. Methodological recommendations cited in the article are an important tool for improving methods of scientific methodical solution of equations involving whole and fractional numbers for students of secondary schools and academic lyceums. In addition, this topic can be used in the improvement

of educational programs and in the development of educational manuals and textbooks in mathematics.

Keywords: whole and fractional part of a number, definition, equation, inequality, problem and solution

Ta‘rif. Biror *a* sonini olib, $x \le a$ tengsizlikni ko‘raylik. Tengsizlikning yechimi $(-\infty; a]$ oraliqdan iborat bo‘lib, bu oraliqdagi eng katta butun sonni [*a*] ko‘rinishda belgilab olamiz va uni *a* sonning butun qismi deb ataymiz.

Masalan, [7]=7; [2]=1; [0]=0; [0,3]=0, [-1]=-1, [-2,3]=-3 ekanini tekshirish qiyin emas. Demak, aniqlanishiga ko‘ra [*a*] albatta, butun son. Berilgan *a* sonning kasr qismi esa, *a-[a]* ifodaga teng bo‘lib, u {*a*} ko‘rinishda belgilanadi. (N.Raximov B. , February 2023.)

Masalan, {5,7}=0,7; {2}=0; {0}=0; {-0,2}=0,8; $\{-\frac{17}{5}\} = \frac{3}{5}$.

Ta‘rifga ko‘ra, $0 \le \{a\} < 1$ tengsizlik o‘rinli, [*a*] esa butun son. Quyidagi xossalar o‘rinli:

a) $0 \le \{x\} < 1$.

b) *x=[x]+{x}*.

c) *[x]=n* bo‘lsa, $n \le x < n+1$ tengsizlik bajariladi.

d) *[x+1]=[x]+1*, umuman, ixtiyoriy n butun son uchun [x+n]=[x]+n.

e) *{x+1}={x}*, ixtiyoriy butun uchun {x+n}={x}.

f) *[{x}]=0; {[x]}=0; [[x]]=[x]; {{x}}={x}*.

g) *{x}={y}* bo‘lsa, *x-y* ayirma butun son bo‘ladi.

h) Ixtiyoriy *n* butun son uchun $[x] > n \Rightarrow x \ge n+1,\ [\mathrm{x}] \ge \mathrm{n} \Rightarrow \mathrm{x} \ge \mathrm{n}$ munosabatlar o‘rinli.

i) $\forall x \in R$ da $\mathrm{x} \ge [\mathrm{x}]$.

Demak, ixtiyoriy *x*-haqiqiy sonni $x = [x] + \{x\}$ ko‘rinishda ifodalab olish mumkin, bu yerda $[x]$ - son *x* -ning butun qismi, $\{x\}$ - son *x* - ning kasr qismi.

Bunda *x* soni $[x] \le x < [x] + 1$ oraliqda, *x* ni kasr qismi esa $0 \le \{x\} < 1$ oraliqda bo‘ladi.

Kiritilgan *[x]* va *{x}* tushunchalar bilan yaqinroq tanishish uchun bir nechta masalalar yechimlarini qarab o‘tamiz.

1-masala. $1 \cdot 2 \cdot 3 \cdot 4 \cdot ... \cdot 2020$ ko‘paytma nechta nol bilan tugaydi.

Yechim. Berilgan ko‘paytmaning kanonik shakli $2^{a_1} \cdot 3^{a_2} \cdot 5^{a_2} \cdot ... \cdot p^{a_n}$ bo‘lsin. Bundan a_1 va a_3 larni topamiz.

$$a_1=\left\lfloor\frac{2020}{2}\right\rfloor+\left\lfloor\frac{2020}{4}\right\rfloor+\left\lfloor\frac{2020}{8}\right\rfloor+\left\lfloor\frac{2020}{16}\right\rfloor+\ldots+\left\lfloor\frac{2020}{1024}\right\rfloor=2013$$

$$a_3=\left\lfloor\frac{2020}{5}\right\rfloor+\left\lfloor\frac{2020}{25}\right\rfloor+\left\lfloor\frac{2020}{125}\right\rfloor+\left\lfloor\frac{2020}{625}\right\rfloor=404+80+16+3=503$$

$1\cdot2\cdot3\cdot4\cdot\ldots\cdot2020=2^{2013}\cdot5^{503}\cdot A=B\cdot10^{503}$. Hisoblash natijada hosil bo‘lgan son 503 ta nol bilan tugaydi.

Bilamizki, *a* sonini tub yoki murakkab son ekanligini *a* ni $[\sqrt{a}]$ dan kichik bo‘lgan tub sonlarga bo‘lib topamiz. Agar *a* son $[\sqrt{a}]$ dan kichik bo‘lgan birorta tub songa bo‘linmasa, bu holda *a* tub son bo‘ladi.

2-masala. 1789 sonini tub son ekanligini aniqlash uchun uni ketma-ket 2, 3, 5 va hokazo tub sonlarga bo‘lib boriladi. Qanday tub songa yetganda bo‘lishni to‘xtatish mumkin? A) 41 B) 43 C) 47 D) 59

Yechim. $\lfloor\sqrt{a}\rfloor=\lfloor\sqrt{1789}\rfloor=[42,297]=42$ ekanligidan, 1789 sonini tub yoki murakkab son ekanligini aniqlash uchun 42 dan kichik eng katta tub songacha bo‘lib xulosa qilish kerak. Demak, 41 gacha bo‘lish kerak. Javob: 41 (A).

3-masala. Tenglamani yeching: $[x]=3x+1$

Yechim. $[x]=n,$ (bunda $n\in Z$) belgilash olamiz. U holda, *3x+1=n* bo‘lib, bundan $x=\frac{n-1}{3}$ ekanini topamiz. Yuqoridagi $[x]\le x<[x]+1$ tengsizlikka asosan $n\le\frac{n-1}{3}<n+1$ tengsizlikni yoza olamiz. Bu esa $\begin{cases} n\le\frac{n-1}{3} \\ \frac{n-1}{3}<n+1 \end{cases}$ tengsizliklar sistemasiga teng kuchli. Tengsizliklar sistemasini yechib, $-2<n\le\frac{-1}{2}$ yechimga ega bo‘lamiz. Bu oraliqdagi butun son *n*=-1 bo‘ladi. U holda, $x=\frac{n-1}{3}=\frac{-1-1}{3}=-\frac{2}{3}$. Javob: $x=-\frac{2}{3}$. [6]

4-masala. Tenglamani yeching. $2[x]=x+2\{x\}$

Yechim. $x=[x]+\{x\}$ ekanligidan:

$$2[x]=x+2\{x\}\Rightarrow 2[x]=[x]+3\{x\}\Rightarrow [x]=3\{x\}$$

$0 \le \{x\} < 1$ ekanligidan: $[x] = 3\{x\} < 3 \Rightarrow \begin{cases} [x] = 0,\ 1,\ 2 \\ \{x\} = 0,\ \frac{1}{3},\ \frac{2}{3} \end{cases}$ ekanligini aniqlaymiz. U holda, tenglamaning yechimi $x = [x] + \{x\} = 0,\ 1\frac{1}{3},\ 2\frac{2}{3}$. Javob: $\left\{0,\ 1\frac{1}{3},\ 2\frac{2}{3}\right\}$.

5-masala. Tenglamani yeching. $7x - 4[x] = 3\{x\} + 8$

Yechim. $x = [x] + \{x\}$ ekanligidan: $7([x] + \{x\}) - 4[x] = 3\{x\} + 8$ yoki $\{x\} = \frac{8 - 3[x]}{4}$. Ma'lumki, $0 \le \{x\} < 1$ ekanligidan, $0 \le \frac{8 - 3[x]}{4} < 1$ bo'lib, bundan $\frac{4}{3} < [x] \le \frac{8}{3}$ kelib chiqadi. $[x]$-butun son ekanligidan $[x] = 2$ ekani ma'lum bo'ladi. U holda, $\{x\} = \frac{8 - 3[x]}{4} = \frac{8 - 6}{4} = \frac{1}{2}$. Demak, $x = [x] + \{x\} = 2 + \frac{1}{2} = \frac{5}{2}$. Javob: $x = \frac{5}{2}$.

6-masala. Tenglamani yeching. $[x - 1] = \left\lfloor \frac{x + 2}{2} \right\rfloor$.

Yechim. $[x - 1] = n$ deb olamiz, u holda $\left\lfloor \frac{x + 2}{2} \right\rfloor = n$ bo'ladi. Bu tengliklardan $\begin{cases} n \le x - 1 < n + 1 \\ n \le \frac{x + 2}{2} < n + 1 \end{cases}$ sistemani yoza olamiz. Bunda ikkita holat bo'lishi mumkin.

1) Aytaylik $x - 1 \ge \frac{x + 2}{2}$ bo'lsin. Bu tengsizlikni yechib $x \ge 4$ natijani olamiz. Bundan $n \le \frac{x + 2}{2} \le x - 1 < n + 1$ kelib chiqadi. U holda, quyidagi tengsizliklar sistemasini tuzish mumkin: $\begin{cases} x \ge 4 \\ \frac{x + 2}{2} \ge n \\ x - 1 < n + 1 \end{cases}$ bundan $x \ge 4,\ 2n - 2 \le x < n + 2$ ni yoza olamiz. Natijada $2n - 2 < n + 2,\ 4 \le x < n + 2$ bo'ladi. Bu tengsizliklardan $2n - 2 < n + 2,\ 4 < n + 2$ bo'lib, natija $2 < n < 4$ tengsizlikka kelamiz n-butun son ekanligini inobatga olsak, $n = 3$ kelib chiqadi. U holda, $x \ge 4,\ 2n - 2 \le x < n + 2$ dan $4 \le x < 5$ natijami olamiz.

2) $x - 1 < \frac{x + 2}{2}$ holatni qarasak, $x < 4$ bo'ladi. Bunda ham $n \le x - 1 \le \frac{x + 2}{2} < n + 1$ tengsizlikni 1-holdagi kabi tahlil qilib $3 \le x < 4$ natijani olamiz. Javob: $[3;5)$.

7-masala. Tenglamani yeching. $x^2-8[x]+7=0$.

Yechim. Berilgan tenglamaning ildizi *x* bo‘lsin. $n=[x]$ deb olsak, $x^2+7=8n$ bo‘ladi. Demak, *n*>0. Endi $n=[x]$ belgilashdan $n\le x<n+1$ ni yoza olamiz. Oxirgi hosil bo‘lgan tengsizlikni kvadratga ko‘tarib 7 ni qo‘shsak, $n^2+7\le x^2+7<(n+1)^2+7=n^2+2n+8$ hosil bo‘ladi, $x^2+7=8n$ ekanligini e’tiborga olsak $n^2+7\le 8n<n^2+2n+8$ tengsizlikni hosil qilamiz. Bu tengsizlikka teng kuchli quyidagi tengsizliklar sistemasini yozib, yechimni topsak:

$$\begin{cases} n^2+7\le 8n \\ 8n<n^2+2n+8 \end{cases} \Rightarrow \begin{cases} 1\le n\le 7 \\ n<2 \text{ va } n>4 \end{cases} \Leftrightarrow 1\le n<2 \text{ yoki } 4<n\le 7$$

Oxirgi natijadan va n-butun son ekanligidan *n*=1, 5, 6, 7 holatlar bo‘lishi mumkin.

Bu qiymatlarni navbatma-navbat $x^2+7=8n$ ifodaga qo‘yib *x* ning qiymatlarini topamiz(bunda $n\le x<n+1$ ekanidan *x*>0). Javob: $1;\ \sqrt{33};\ \sqrt{41};\ 7$. (N.Raximov B. , Methods of solving equations related to whole and fractional part of a number. , Volume 14, November, 2022y)

8-masala. $\int_0^2 [x^2]dx$ aniq integralni hisoblang.

Yechim. Dastlab masala shartida berilgan integralni oraliqlarga ajratib olamiz:

$$I=\int_0^2[x^2]dx=\int_0^1[x^2]dx+\int_1^{\sqrt{2}}[x^2]dx+\int_{\sqrt{2}}^{\sqrt{3}}[x^2]dx+\int_{\sqrt{3}}^{2}[x^2]dx.$$

$$I=\int_0^1 0dx+\int_1^{\sqrt{2}}1dx+\int_{\sqrt{2}}^{\sqrt{3}}2dx+\int_{\sqrt{3}}^{2}3dx=x\,|_1^{\sqrt{2}}+2x\,|_{\sqrt{2}}^{\sqrt{3}}+x\,|_{\sqrt{3}}^{2}=(\sqrt{2}-1)+$$
$$+2(\sqrt{3}-\sqrt{2})+3(2-\sqrt{3})=\sqrt{2}-1+2\sqrt{3}-2\sqrt{2}+6-3\sqrt{3}=5-\sqrt{3}-\sqrt{2}$$

Javob: $5-\sqrt{3}-\sqrt{2}$. (N.Raximov, Dekabr, 2023.)

9-masala. Tengsizlikni yeching: $x^2-4[x]+3\ge 0$, bunda $[x]$- *x* ning butun qismi.

Yechim. Dastlab biz bu tengsizlikka teskari tengsizlikni qaraymiz. $x^2-4[x]+3<0$ tengsizlikni qanoatlantiruvchi barcha x larni topamiz. $n=[x]$ deb olamiz. U holda $x^2+3<4n$ bo‘lib, bundan *n*>0 ekani ma’lum. $n=[x]$ dan $n\le x$. Demak, $n^2+3\le x^2+3<4n$ bo‘lib, bundan $n^2-4n+3<0$ ni hosil qilamiz. Bu tengsizlikni yechib 1<*n*<3 yechimga ega bo‘lamiz. *n* -butun son ekanligidan *n*=2 bo‘ladi. $n\le x$ ekanidan $2\le x$ ikkinchi tomondan $x^2+3<4n=8$ bo‘lib, bundan $x<\sqrt{5}$ bo‘ladi. Demak, yechim $[2;\sqrt{5})$ bo‘ladi. Biz berilgan tengsizlikka teskari bo‘lgan tengsizlikni yechdik. U holda, $x^2-4[x]+3\geq 0$ tengsizlikning yechimi $(-\infty;2)\cup[\sqrt{5};\infty)$ bo‘ladi. (N.Rahimov, 2023y)

10-masala. [x]+2{x}=22 tenglamani yeching.

Yechim.

$$[x]+2\{x\}=22\Rightarrow[x]+\{x\}+\{x\}=22\Rightarrow x+\{x\}=22\Rightarrow 0\leq 22-x<1\Rightarrow 21<x\leq 22$$

tengsizlikni hosil qilamiz. Endi *2{x}* butun son ekanligidan $\{x\}=\frac{n}{2},\ n\in Z$ kelib chiqadi. Bundan *x=21,5* va *x=22* yechimlarni olamiz. Javob: $x=21\frac{1}{2}$ va $\mathrm{x}=22$

11-masala. Agar $[x]\cdot\{x\}=100$ bo‘lsa, u holda $[x^2]-[x]^2$ ifodaning qiymatini toping.

Yechim. Yuqoridagi d) va f) xossalaridan foydalanamiz:

$$[x^2]-[x]^2=[([x]+\{x\})^2]-[x]^2=[[x]^2+2[x]\{x\}+\{x\}^2]-[x]^2=[200+\{x\}^2]=200+[\{x\}^2]=200$$

Javob: 200. (N.Raximov B. , Maktab o‘quvchilarida sonning butun va kasr qismiga oid masalalarni yechich ko‘nikmasini shakllantirish. , 2022/3/26.)

12-masala. Tenglamani yeching. [x+2]+[x+3]-[x+4]=3.

Yechim. Yuqoridagi d) xossadan foydalanamiz.

$$[x]+2+[x]+3-[x]-4=3\Rightarrow[x]=2\Rightarrow 2\leq x<3$$ Javob: $2\leq x<3$.

Foydalanilgan adabiyotlar

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