Strictly non-volterra dynamic system about dynamics

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Abstract: The article examines the initial problems that led to the creation of dynamic systems. Numerical solutions are found for a time-continuous analogue of a fixed non-Volterra quadratic stochastic operator with discrete time. Using the MathCAD mathematical package, numerical solutions were determined for various initial values and parameters, and graphs and phase trajectories of the solutions were constructed. The advantages and disadvantages of the Runge-Kutte method used in solving the problem are highlighted.

Keywords: population, Fibonacci numbers, population structure, quadratic stochastic operator, numerical solution, continuous-time analog of a dynamic system, phase space, fixed point

Introduction. The development of living things manifests itself differently in different processes. It is influenced by birth, growth, individuality, death of individuals, external environment, and so on. Taking these circumstances into account, a mathematical model of a biological population is constructed.

The number of the population changes and dynamics (population dynamics is a change in its biological and group properties over time. The most important characteristics for observing population dynamics are such characteristics as changes in the number of individuals, their biomass, as well as changes in age and sex structure) represents. Population dynamics is a branch of mathematical biology and is a field that focuses on determining the state of a population in time. Because, if every process is mathematically modeled, this modeling gives an opportunity to get complete information about the studied process, makes it possible to draw conclusions about the future situation.

The setting of mathematical problems for the study of biological processes (population) dates back to ancient times. Indeed, it is important to study such issues and draw conclusions.

The first research on the mathematical model of the biological population is presented in the work of Leonardo Fibonacci, who lived in 1170-1240, «Treatise on counting» («Liber abaci»).

This book, which is a collection of arithmetic and algebraic information, deals with the following problem, which was widespread at that time and later throughout Europe: «how many rabbits are born from a pair of rabbits in a year, if two months after the birth of a pair of rabbits, one rabbit is born from them». The solution to this problem consists of the following numbers:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, ...

These numbers went down in history as Fibonacci numbers.

In real populations, reproduction and mortality rates vary among different groups. For example, insects lay eggs and their enemies kill the larvae, in addition, they are affected by environmental metabolic products, cannibalism and poisoning, age stages and their intensity. In other words, population dynamics are mainly affected by birth rate, mortality, and population growth rate: birth rate is the number of new individuals appearing per unit of time as a result of reproduction; mortality - the number of individuals who died in a population per unit of time; population growth rate - change in population size per unit of time.

In turn, the analysis of the dynamics of the biological population is important both theoretically and practically. Methods of controlling the reproduction of pest insects (releasing sterile male insects, pheromone traps, etc.) are aimed at creating a certain imbalance in the population structure, which slows down their reproduction and leads to its destruction. These issues are one of the urgent issues of epidemiology.

Quadratic stochastic operators are used to solve problems that arise in a biological population. Quadratic operators attract the attention of specialists in various fields of mathematics and its applications (see, for example, [1]-[2]). The concept of a quadratic stochastic operator was first formulated in article [3]. In the work of S. Ulam [4], the problem was posed of studying the behavior of trajectories of quadratic stochastic operators. This problem is mainly solved for Volterra operators (see [6]-[7]. But the class of non-Volterra operators has been little studied. In this paper, we study one particular case of a continuous analogue of a quadratic stochastic operator [8], which we will call strictly non-Volterra. Note that the class of strictly non-Volterra operators is a subclass of non-Volterra operators. Since there is no general theory studying such operators, it is natural to first study their subclasses.

The motivation for considering arbitrary quadratic operators can be found, for example, in [5] and [9]. Consequently, each operator is an interesting example in the theory of multidimensional, nonlinear dynamic systems, with diverse behavior of trajectories.

These models, built on the basis of the system of ordinary differential equations, do not give positive results (since the reproduction process is discrete) in expressing the dynamics of seasonal reproduction of many other species.

Mathematical models of these types of processes, represented by a system of simple differential equations with momentum, are the most suitable models and represent the studied process close to the real one. As we mentioned above, the mathematical model of the evolution of populations in biology is represented by quadratic stochastic operators [3].

The problem of finding a limited distribution of individuals of different types during the evolution of a biological system is equivalent to studying the asymptotic properties of a quadratic stochastic operator. In addition, the number of simple and non-standard problems and unsolved problems in the theory of quadratic stochastic operators are of great interest from a mathematical point of view.

The non-linearity of operators, the presence of complex and difficult calculations in the study of trajectories, the lack of widespread development of methods for finding analytical solutions and the need to perform a large number of calculations when studying quadratic operators did not generate interest in solving these types of problems. However, as a result of the advent of computers, interest in the problem of studying the state of the trajectories of quadratic stochastic operators has been revived. Ulam and his colleagues did quite a bit of computing on learning quadratic operators.

In [8], the discrete-time case of a strictly non-Volterra dynamical system is studied. Thus, in this work the uniqueness of the fixed point is proved. This fixed point has been proven to be non-attractive. A description of the $\omega -$limit set of a trajectory for some subclasses of such operators is given. It is shown that, unlike Volterra operators, strictly non-Volterra operators have cyclic trajectories. For two specific operators, it is proved that there exists a cyclic trajectory with period 3, and any trajectory starting on the boundary of a simplex converges to this cyclic trajectory, and trajectories with an initial point (not fixed) lying inside the simplex diverge; The $\omega -$limit set of such a trajectory is infinite and lies on the boundary of the simplex. Subclasses of strictly non-Volterra operators are also studied, the trajectories of which in the limit tend to a cyclic trajectory with period 2.

The main part. In this paper, a continuous-time analogue of a particular case of a discrete-time non-Volterra discrete-time dynamical system studied in [8] a scientific work is studied. The continuous-time analogue of the quadratic stochastic operator studied in the scientific research of U.A. Rozikov and U.U. Jamilov (in general) has the following form:

$$\begin{cases} \dot{x}_1 = \alpha x_2^2 + c x_3^2 + 2x_2x_3 - x_1 = f_1(x_1, x_2, x_3), \\ \dot{x}_2 = a x_1^2 + d x_3^2 + 2x_1x_3 - x_2 = f_2(x_1, x_2, x_3), \\ \dot{x}_3 = b x_1^2 + \beta x_2^2 + 2x_1x_2 - x_3 = f_3(x_1, x_2, x_3) \end{cases} \quad (1)$$

or vector representation of this system

$$x(t) = f\big(x(t)\big), x(t) = \big(x_1(t), x_2(t), x_3(t)\big),$$

$$f\big(x(t)\big) = \Big(f_1\big(x(t)\big), f_2\big(x(t)\big), f_3\big(x(t)\big)\Big),$$

where $x(t)$ is the state of a biological system, $t \geq 0$, $x_1(t) \geq 0, x_2(t) \geq 0, x_3(t) \geq 0$.

We are following the parameters and the solution we are looking for in this article

$$x_1(t) + x_2(t) + x_3(t) = 1, (2)$$

$$\alpha, \beta, a, b, c, d > 0, \beta = b = c + d = 1 \ (3)$$

we study the case that satisfies these conditions.

Main part. Search for fixed points of the system (1), numerical solutions, draw a phase portrait, analyze the obtained results.

It should be said that the special discrete-time case of system (1) with $\alpha = \beta = a = b = c = d = 1/2$ was studied in [10], and the continuous-time case was studied in [11].

A system whose parameters (3) satisfy the condition (1) will be as follows:

$$\begin{cases} \dot{x}_1 = x_3^2/2 + 2x_2x_3 - x_1, \\ \dot{x}_2 = x_3^2/2 + 2x_1x_3 - x_2, \quad (4) \\ \dot{x}_3 = x_1^2 + x_2^2 + 2x_1x_2 - x_3. \end{cases}$$

For convenience, we omit the arguments of the functions.

Definition. Points of the phase space satisfying $f(x^*(t)) = 0$ are said to be fixed points of system (1). Obviously, $x^*(t)$ −itself is a solution of equation (1), because $\dot{x}^*(t) = 0$.

To find the fixed points of the system (4), If we use

$$x_1 + x_2 + x_3 = 1$$

we determine that the system (4) has the following fixed points [8]:

$$x_1^* = \frac{(7 - 3\sqrt{6})/2 + 4\sqrt{5} - 8}{2(4 - \sqrt{5})}, x_2^* = \frac{(3\sqrt{5} - 7)/2 + \sqrt{5} - 1}{2(4 - \sqrt{5})},$$

$$x_3^* = \frac{3 - \sqrt{5}}{2}. (5)$$

That the system (4) has a unique fixed point $M(x_1^*, x_2^*, x_3^*)$ and the type of fixed point is detailed in [8].

As can be seen from the system (4), the first fixed point of the system is $M_0(0,0,0)$. This fixed point cannot be an equilibrium state for the system (4), that satisfies the conditions of the problem under study. The fact that the point $M_0(0,0,0)$ cannot be an equilibrium state for the system (4) satisfying the condition of the problem is explained in detail in the article [12] in three different ways. Here is one method: $x_1 + x_2 + x_3 = 0$. Considering that x_1, x_2, x_3 are probabilities and $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$ satisfy the inequality, the solution of the equation is $x_1 = x_2 = x_3 = 0$, which does not satisfy condition (2). Therefore, $X = 0$ cannot be a solution to the main problem.

We consider the Cauchy problem for system (1). Let it be, at $t = 0$

$$x_1(0) = x_1^0, x_2\,(0) = x_2^0, x_3(0) = x_3^0 \quad (6)$$

The initial conditions $(x_1^0, x_2^0, x_3^0)^T$ are taken from the values corresponding to the columns of matrix B.

$$B = \begin{pmatrix} 0.1 & 0.2 & 0.33 & 0.2 & 0.44 & 0.23 & 0.55 & 0.3 & 0.5 & 0.01 \\ 0.3 & 0.4 & 0.1 & 0.7 & 0.22 & 0.4 & 0.44 & 0.3 & 0.4 & 0.08 \\ 0.6 & 0.4 & 0.57 & 0.1 & 0.34 & 0.37 & 0.01 & 0{,}4 & 0.1 & 0.91 \end{pmatrix}$$

in this $B = (x_1^0, x_2^0, x_3^0)^T$. Numerical solutions of system (4) satisfying these initial conditions were found using MathCAD mathematical package. Graphs are presented in Figures 1-3.

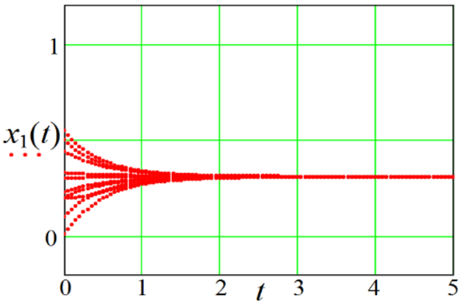

Figure 1

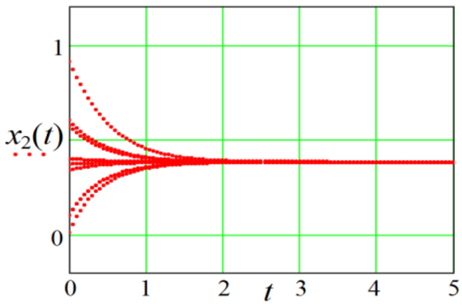

Figure 2

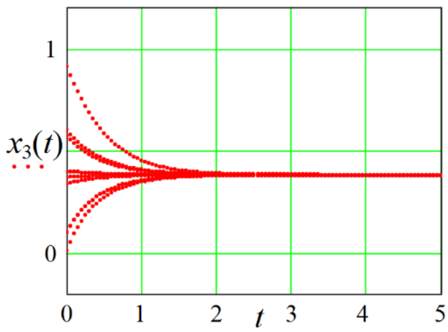

Figure 3

Charts 1-3 show graphs of numerical solutions of system (4) satisfying initial conditions (6) through $x_1(t), x_2(t), x_3(t)$.

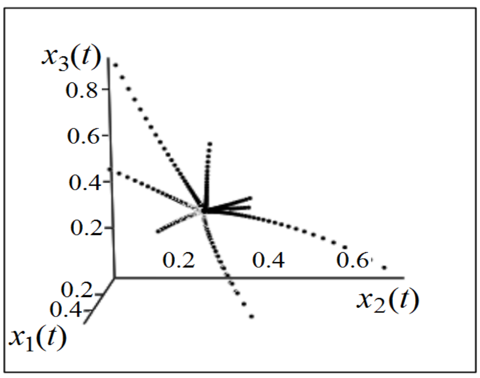

Figure 4

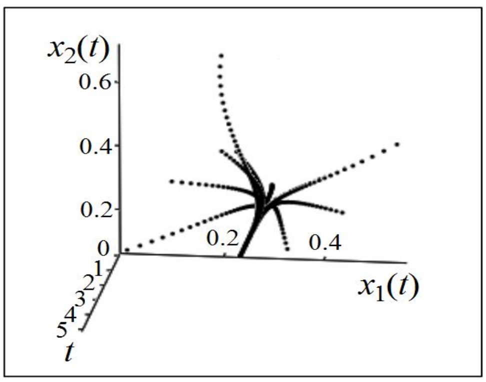

Figure 5

Figure 4-5 shows the phase trajectories of numerical solutions $(x_1(t), x_2(t), x_3(t))$ and $\big(t, x_1(t), x_2(t)\big)$ of the system (4) satisfying the initial conditions (6) described.

Figure 6 depicts the space-phase trajectory of numerical solutions $(t,, x_2(t), x_3(t))$ of system (4) satisfying the initial conditions (6).

Note that in [13], to study system (4), the explicit Euler method was used to numerically solve a system of ordinary differential equations (Cauchy problem). For various values, numerical solutions (2) were obtained using C++ programming.

However, graphs of solutions and phase trajectories are not provided. In this article, using the mathematical package MathCAD, numerical solutions of the system were found. The fourth-order Runge-Kutte method was used to find numerical solutions [14-32].

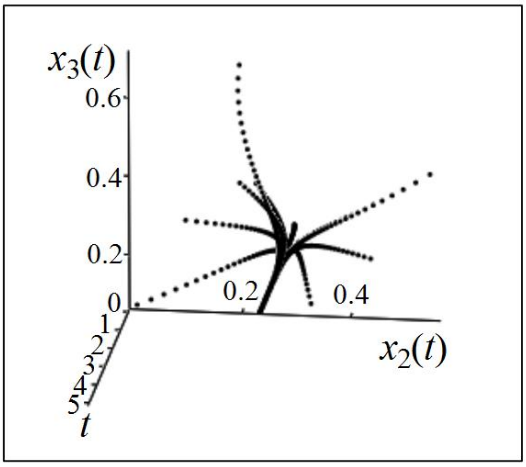

Figure 6

Summary. Here, it should be said that, the explicit Runge-Kutte method was used to solve the problem. Unfortunately, explicit Runge-Kutta methods are, as a rule, unsuitable for solving rigid systems of equations due to the small region of their absolute stability. The instability of explicit Runge-Kutta methods creates very serious problems in the numerical solution of partial differential equations.

If we analyze the solutions, starting from $t \geq 3$, it is observed that the solutions of the system (4), satisfying the initial conditions (5) tend to the fixed point. Here, using the methods used in the preparation of the article and the results obtained in the article [8], the following hypothesis can be stated.

Hypothesis. Some solutions of system (4) tend to the fixed point in (5) at different initial values at $t \geq 3$.

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