The method of finding extrema of functions of several variables

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Abstract: The extrema of the functions are the maximum or minimum price of the function. Each function has a graph in which we can see the maximum and minimum points and by using different methods, those points can be obtained accurately. Extrema are among the important points of the function because at these points, the function takes its maximum and minimum value. The importance of this issue is in receiving the highest or lowest price. When we consider the gasoline consumption function of Farah province, by getting the extremes of this function, we can determine how the effective factors in this function change in order to have the lowest amount of consumption and minimize this function. Similarly, any function can be considered for any problem and make it maximal or minimal. Therefore, the purpose of this research is to get the extrema of the functions so that the provisions and losses of the companies can be calculated more accurately. This research method is a phishing library and authentic scientific books have been used. As a result, every company works based on a function of several variables, after receiving its extremes, it is possible to understand how much the variables are so that the company has the most benefit and at the same time the least loss. So, the methods identified in this research to obtain the extremes of the functions of several transformations can play an effective role in improving the activities of industrial companies.

Keywords: minimum, maximum, extreme, multiple, contradictory.

INTRODUCTION

The development and progress of science and technology in today's era is progressing at such a speed that in a very short period of time we are witnessing remarkable innovations and developments. Mathematics appeared in the human world since the birth of man and died They realized that they urgently need this knowledge. In the life of the new age, mathematics answers most of the human needs and is very important in the form of modern systems. The basis and foundation of all the current developments are the laws and phenomena that have been used in practice for many years, and in order to evolve them as much as possible, continuous and tireless research and basic studies are going on. All companies work on the basis of functions, in which several branches of mathematics are used, each of which has its own importance. Among them, mathematical analysis, which has received more attention in recent centuries, is considered one of the most important parts of mathematics and is considered to be a basic principle and a major goal with many applications for many subjects. Extremes are a very important part of mathematical analysis and are used to get the highest or lowest price of the function, based on which the highest profit or the lowest loss of a company can be accurately calculated. In this research, in order to measure the maximum benefit and the minimum loss of an industrial company or even a small company, appropriate solutions have been received so that it can be understood how the variables of a company's function change and finally by mathematical operations, especially extremes, function, To receive the maximum benefit and the minimum loss of the company. So this subject has a special need for research and plays a very important role in life because all our daily work is based on functions and all functions have graphs and each graph has extreme points. In this research, an effort has been made to firstly identify the extrema of the functions of several transformations in a precise manner, and secondly, to fully understand the ways of obtaining the extrema so that those interested can use it correctly. Students of mathematics, physics, chemistry, biology, computer and engineering fields can use this research as a solution to their needs. So, with all the importance mentioned above, research on this issue is important and can play a significant role in advancing the affairs of companies and factories and even in everyday life.

Function: We consider two sets A and B, If to each element of the set, according to a special rule, only one element (and not more) of the elements of the set B is associated, In this case, they say that a function of A is defined in B and they write f:A→B and read f is a function of A in B(Pourkazmi,2003:45).

Functions of several variables: Now we extend the concept of function to the transformed function n. The full discussion of these topics belongs to advanced calculus. Here, our discussion of functions with more than one variable is exclusive to two- and three-variable functions, however, we provide definitions for n-variable functions and then we show the application of these definitions for two- and three-variable functions. Also, we show that when any of these definitions is applied to a function of a transform, the previous definition is obtained.

Definition: A function n is a transform set of ordered forms of the form (p, w) where no two distinct ordered forms have the same first element. p is a point in the n-dimensional number space and w is a real number (Litthold, 2010).

Domain of functions several variables

The set of all possible values of p is called the domain of the function and the set of all possible values of w is called the codomain of the function. From this definition, it is clear that the domain of a function n transform is the set of points in $\mathbb{R}^n$ and its codomain is the set of real numbers or the set of points in $\mathbb{R}^1$. When $n = 1$, we have a transform function. Because the second and co-second of that set are real numbers. When $n = 2$, we have a function of two variables, the second of which is a set of $\mathbb{R}^2$numbers or a set of ordered types (x, y) of real numbers, and the codomain is a set of real numbers $(Lithold, 2010)$.

Example: the polynomial function domain is defined below.

$$f(x,y) = \sqrt{x^2 + y^2 - 4}$$
$$D_f = \{(x,y)|x^2 + y^2 - 4 \geq 0\} = \{(x,y)|x^2 + y^2 \geq 4\}$$

The equation $x^2 + y^2 = 4$ is a circle with center (0,0) and radius 2. With a little investigation, it is clear that the second function f is the points outside and on this circle.

Explanation: Suppose that the function f of two variables x and y is the set of all ordered forms in the form (p,z) such that $f(x,y) = z = \sqrt{25 - x^2 - y^2}$ The domain function is the set of all ordered forms (x,y) such that $25 - x^2 - y^2 \geq 0$ .This is the set of all points in the xy plane on the circle $x^2 + y^2 = 25$ and inside the area is limited to this circle. Because $f(x,y) = z = \sqrt{x^2 + y^2 - 4}$ and $0 \leq z \leq 5$ Therefore, the codomain of f is the set of all real numbers It is in the interval [0,5].

Figure (1) shows the set of second f points as shaded areas in R^2

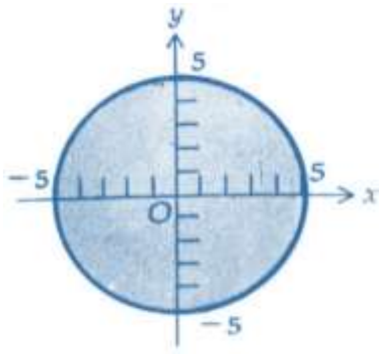


Figure (1) The domaon function $z = \sqrt{x^2 + y^2 - 4}$ (Leit hold,1389)

Graph of functions of several variables

Definition: Whenever f is a n transformed function, the graph of f is the set of all points $(x_1, x_2, \dots, x_n, \mathrm{w})$ at R^{n+1} In which $\mathrm{w} = (x_1, x_2, \dots, x_n)$ is a point in the domain f. For the three-variable function, there is something like the level curves of a two-variable function(James, 2006).

Example: Determine the graph of the function $f(x,y) = \mathrm{x}^2 + \mathrm{y}^2$.

Solution: Some of the function level curves are as follows

$z = 1 \Rightarrow x^2 + y^2 = 1$ $\qquad$ $z = 0 \Rightarrow x^2 + y^2 = 0$,

$z = 9 \Rightarrow x^2 + y^2 = 9$ $\qquad$ $z = 4 \Rightarrow x^2 + y^2 = 4$,

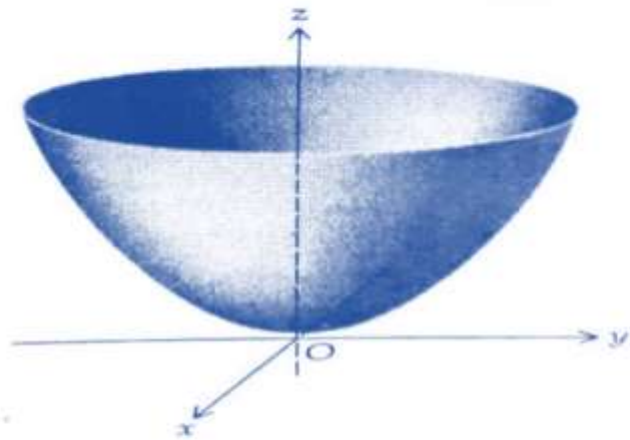


Figure (2) showing the graph of the function $z = x^2 + y^2$

In the equation $x^2 + y^2 = 0$, only the point $(0,0)$ applies But the rest of the curves are aligned to the center $(0,0)$ and the radii are $1, 2$ *and* 3. If we connect the point $(0,0)$ and these circles in space, it is called a rotating paraboloid of shape (2) (Leit hold,2010).

Limit functions of several variables

The concept of limit of functions of several variables is similar to the limit of functions of one variable. To define the limit of the function of several variables, it is enough to slightly adjust the definition of the limit of the function of one variable. The way it works is that, suppose $f(p)$ is a function of several variables defined in the vicinity of $p_0$and we still have: $p_0 = (a_1, a_2, \ldots, a_n)$ and $p = (x_1, x_2, \ldots, x_n)$ We know that $f(p)$ is an abbreviation for $f(x_1, x_2, \ldots, x_n)$ and p_0 is an abbreviation for $\mathrm{f} = (a_1, a_2, \ldots, a_n)$. In this case, it can be said that when p approaches p_0, then $f(p)$ approaches the number l, if $\delta > 0$ can be found for every $\varepsilon > 0$, so that if $0 < |p_0 p| < \delta$ and it results that$|f(p) - l| < \varepsilon$ also applies This matter can be expressed as $\lim_{p \to p_0} f(p) = i$ Or express as p → p_0 ⟹ f(p) → l (Silverman, 2007).

Note: The limit of functions of multiple variables can be calculated with the help of a number of theorems, as long as the limit of functions of one variable is not ambiguous, for the sake of brevity, we refrain from mentioning those theorems and we will remind you of those theorems by mentioning examples(Thomas, 2011).

Example: The following limits are calculated with the help of a number of theorems.

$$\lim_{(x,y)\to(2,3)} 2x + y^2 = 6 + 4 = 10$$

$$\lim_{(x,y)\to(2,1)} x\cos^{-1}(y-1) = 2\cos^{-1}(0) = 2\left(\frac{\pi}{2}\right) = \pi$$

Partial derivatives of n variables

We consider the function $z = f(x_1, x_2, \ldots, x_n) = \mathrm{f(x)}$. Whenever we multiply one of the variables and the other variables remain constant, for example, if we multiply x_i and the other variables remain constant, we have:

$$\begin{cases} x_i \to x_i + \Delta x_i \\ x_j \to x_j, i \neq j \end{cases}$$

$$z_1 = f(x_1, x_2, \dots, x_n) \to z_2 = f(x_1, x_2, \dots, x_i + \Delta x_i, x_{i+1,} \dots, x_n),$$

$$\Delta z_i = z_1 - z_2$$

$$\lim_{\Delta x_i \to 0} \frac{f(x_1, x_2, \dots, x_i + \Delta x_i, x_{i+1,} \dots, x_n) - f(x_1, x_2, \dots, x_n)}{\Delta x_i}$$

If this limit exists, we call it the partial derivative of the function z with respect to the variable x_i and show it as $\frac{\partial z}{\partial x_1}$. Similarly, in the general case, the function z has n partial derivative as follows (Pourkazmi, 2003).

$\frac{\partial z}{\partial X} = (\frac{\partial z}{\partial x_1}, \frac{\partial z}{\partial x_2}, , \dots, \frac{\partial z}{\partial x_n})$

Relative extrema of functions of several variables

A point that is a relative maximum or minimum is called a relative extremum point. Also, if a value is relative maximum or minimum, that value is called relative extremum. The absolute and subjective extrema of the n-variable function $f(x_1, x_2, \dots, x_n)$ are basically the same as the extrema of two variables, with the difference that $(x_1, x_2, \dots, x_n)$ instead of (x, y) and $(a_1, a_2, \dots, a_n)$ is defined instead of (a, b). If for $(x_1, x_2, \dots, x_n)$ in the vicinity of because $\sqrt{(x_1 - a_1)^2 + (x_2 - a_2)^2 + \dots + (x_n - a_n)^2} < \delta$ from $(a_1, a_2, \dots, a_n)$ the relation $f(a_1, a_2, \dots, a_n) \geq f(x_1, x_2, \dots, x_n)$ applies, The vector f in $(a_1, a_2, \dots, a_n)$ has a relative maximum equal to $f(a_1, a_2, \dots, a_n)$. If for $(x_1, x_2, \dots, x_n)$ in the vicinity such that $\sqrt{(x_1 - a_1)^2 + (x_2 - a_2)^2 + \dots + (x_n - a_n)^2} < \delta$ from $(a_1, a_2, \dots, a_n)$ we have $f(a_1, a_2, \dots, a_n) \leq f(x_1, x_2, \dots, x_n)$, We say that f in $(a_1, a_2, \dots, a_n)$ has a relative minimum equal to $f(a_1, a_2, \dots, a_n)$ (Silverman, 2007).

Definition: Let $f(x, y)$ be a function and $(a, b) \epsilon D_f$ if there exists a circle with center (a, b) that for each (x, y) belongs to this circle:

a) If $f(x, y) \geq f(a, b)$, then $f(a, b)$ is called the relative minimum and the point (a, b) is called the relative minimum point of the function f.

b) b) If $f(x, y) \leq f(a, b)$, then $f(a, b)$ is called the relative maximum and the point (a, b) is called the relative maximum point of the function f.

c) c) If the previous inequalities are valid for all the values of D_f, instead of the word relative maximum or minimum, maximum and absolute minimum are used(Talahbaz, 2014).

Example: for the function $f(x, y) = (x - 2)^2 + (y - 1)^4$ the point $(2,1)$ is a relative minimum. To demonstrate this claim, we consider an arbitrary circle centered at $(2,1)$. For each point (x, y) of this circle we have.

$$\left.\begin{matrix} (x-2)^2+(y-1)^4+5 \geq 5 \\ f(2,1)=5 \end{matrix}\right\} \to f(x, y) \geq f(2,1)$$

Therefore, the function at the point (2,1) has a relative minimum (kerayechian, 2010).

The theorem is a necessary condition for the existence of relative extrema

We have a function with the equation $z = f(x, y)$ and this function is definite and polynomial in (x_0, y_0) and its neighborhood, and have relative maximum or minimum at (x_0, y_0) and there are f_x and f_y at this point, in this case these partial derivatives are equal to zero at this point, that is,

$$\begin{cases} f_x(x_0, y_0) = 0 \\ f_y(x_0, y_0) = 0 \end{cases}$$

Note: From the solution of the above system of equations, the coordinates of points such as (x_0, y_0) can be obtained, and the corresponding z_0 value can be obtained in the input function for example, (x_0, y_0, z_0) It is determined. This point is the critical point $z=f(x,y)$. This point may be maximum or minimum or none. It should be noted that the above theorem can be generalized for the variable n function, that is, if $z = f(x_1, x_2, \ldots, x_n)$, the necessary condition for the existence of maximum and minimum is that we have,

$$z_{x_1} = 0\,, z_{x_2} = 0\,, \ldots, z_{x_n} = 0$$

Critical points of functions of several variables

We consider the function $z = f(x_1, x_2, \ldots, x_n)$, the points where

$f_{x_1} = f_{x_2} = f_{x_n} = 0$ or the points where one of the first order partial derivatives of this function does not exist, are called critical points of the function. for $n = 3$, we usually write $z = f(x, y, z)$ instead of $z = f(x_1, x_2, \ldots, x_n)$ and instead of $(a_1, a_2, \ldots, a_n)$ we write (a,b,c) a critical point is f, so if $f(x, y, z)$ is a relative extremum in (a, b, c), then (a, b, c) is a critical point of f. For the bivariate function, the following definition can be considered ($Silverman,$ 2007).

Definition: For the function $f(x, y)$, the point $(a, b) \epsilon D_f$ is called a critical point, if one of the following is true:

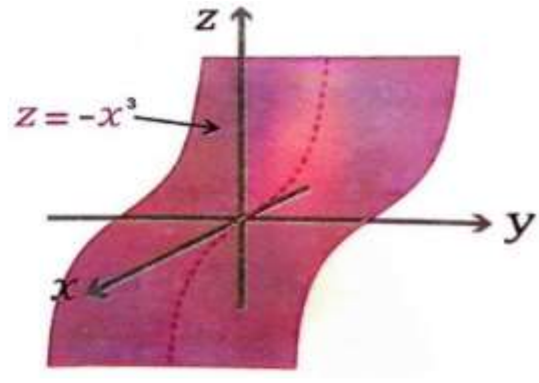
a) f_x (a,b)= f_y (a,b)=0

b) At least one of the two values of f_x (a, b) and f_y (a, b) is not available

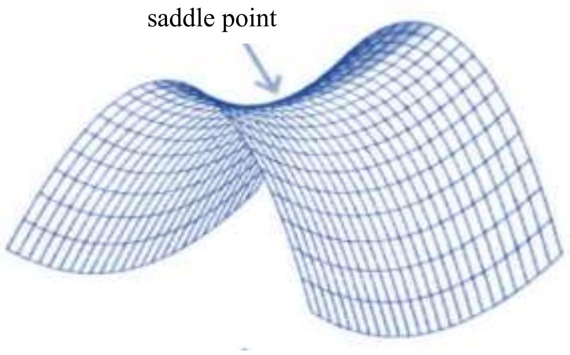
Definition: For the function $z = f(x, y)$, we call the point $(a, b)\epsilon D_f$ a saddle point of the function if we have:

$$f_x(a,b) = f_y(a,b) = 0$$

and this extreme point should not be relative(Ghori, 2009).



Figure(a



Figure(b

Figure (3) saddle point for the function $z = x^2 - 4y^2$ (kerayechian, 1934)

CONCLUSION

The extremes are the maximum and minimum points of the function; It means the highest and the lowest point of the graph of the function or we can say that the extremes are the points of the graph of the function that show at which points the function nction has the highest value and in

Which points have the lowest value. The extrema are studied in two forms, relative and absolute, and after we have received the critical points of the function, by using relations such as Δ test, sign d^2z , Hessian determinant, etc., we can receive the extrema of the functions and recognize its nature, that is, whether it is a maximum or a minimum.

The limit is the approximation of the function to a certain point when the transform of that function approaches a certain point. The derivative is the limit of the ratio of changes of the function to the variable increment when the variable increment approaches zero. Continuity is the continuity of the function at a certain point. Understanding the concepts of limit, derivative and metamateriality is necessary to get the extrema of the functions of several variables. When we have the profit function and the loss function of an industrial company, we can get the extrema of those functions by using the relationships that have been clarified in detail, and later we can know how the transformations of the profit and loss functions of this company will change or how much to maximize the benefit of the company and minimize the loss of the company.

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