Application area of conical sections of the parabola in the area of light reflection

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Abstract: This work, structured in two main parts, is devoted to the parabola topic. In the first, the elements of a parabola are reviewed by linking the definition of intersection of a plane and a cone with a locus of the plane. The necessary part for calculation of the elements as a curve in space are pointed out as well as historical notes and properties of light reflection are included. In the second part, the applications of parabola conic in many parts of physics. A project based on the reflection of light is proposed that seeks to connect various subjects in line with the new educational paradigm of development of key competencies, joining different fields of knowledge.

Keywords: Parabola, Reflection, Focal chord, Latus rectum and Light focus

INTRODUCTION

Apollonius of Pergo (3rd century BC and 2nd century BC) studied conics, demonstrating these curves are obtained by sections through a plane to any cone, not necessarily with an angle at the vertex of $\frac{\pi}{2}$ rad. It is not clear that Apollonius knew the importance of the directrix of the parabola [4]. Apollonius also gave the definition of a conical surface as that which is generated when a straight line remains fixed at one point while, through another point, it moves along a fixed circumference. The point that remains fixed is called the vertex and the line that passes through the vertex and the center of the circle is called the axis of the cone fig. 1. Menecmus (4th century BC) worked on parabolic curves as a tool to solve the problem of doubling the cube [2]. In this problem, given a real number $p > 0$, there is a hexahedron of volume p^3, the question is what the length of the edge will be, if we want to build a hexahedron with twice the volume, that is, $2p^3$. Clearly the length is $\sqrt[3]{2p}$ but it is:

$$\frac{x}{2p} = \frac{y}{x} = \frac{2p}{y}$$

The solution (x_0, y_0) is the intersection point of the curves $y^2 = 2px$, $x^2 = 2py$. Even more, $x_0 = \sqrt[3]{2p}$. Therefore, finding $\sqrt[3]{2p}$ is possible if both curves can be traced. Arquimedes de Syracuse (287 BC-212 BC) took a keen interest in the parabola and dedicated the book entitled The quadrature of the parabola to it [5].

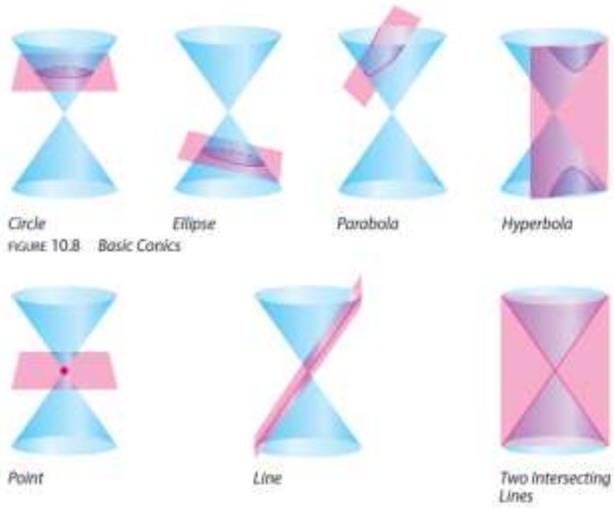


Fig.1[8]

A line segment that passes through the focus of a parabola and has endpoints on the parabola is called a focal chord**.** The specific focal chord perpendicular to the axis of the parabola is called the latus rectum**.** Parabolas occur in a wide variety of applications. For instance, a parabolic reflector can be formed by revolving a parabola around its axis. The resulting surface has the property that all incoming rays parallel to the axis are reflected through the focus of the parabola. This is the principle behind the construction of the parabolic mirrors used in reflecting telescopes. Conversely, the light rays emanating from the focus of a parabolic reflector used in a flashlight are all parallel to one another, as shown in Fig. 2. A line is tangent to a parabola at a point on the parabola if the line intersects, but does not cross, the parabola at the point. Tangent lines to parabolas have special properties related to the use of parabolas in constructing reflective surfaces [1].

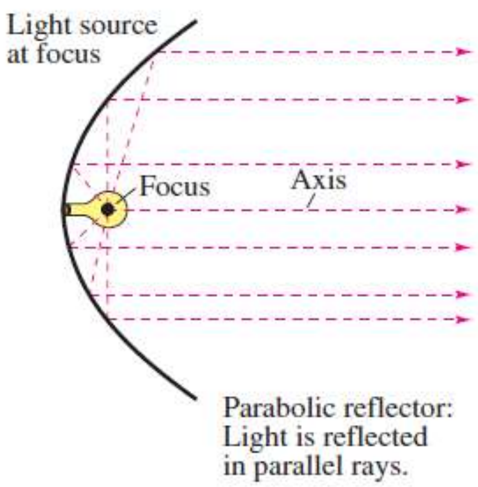


Fig. 2

For that the Parabola Brings Light to a Focus it we'll now look at an infinitesimal segment of the parabola. In Fig. 3 we've shown a highly magnified view of two points on the parabola, marked p_1 and p_2, along with the lines leading from the focus and directrix to them. The segment shown in the picture is so small, and p_1 and p_2 are so close together, that the lines from the focus to p_1 and p_2 are parallel. The lines perpendicular to the directrix which lead to p_1 and p_2 are, of course, necessarily parallel. The proof is contained entirely in the Fig. 3. We will, however, discuss it in a bit more detail, below.

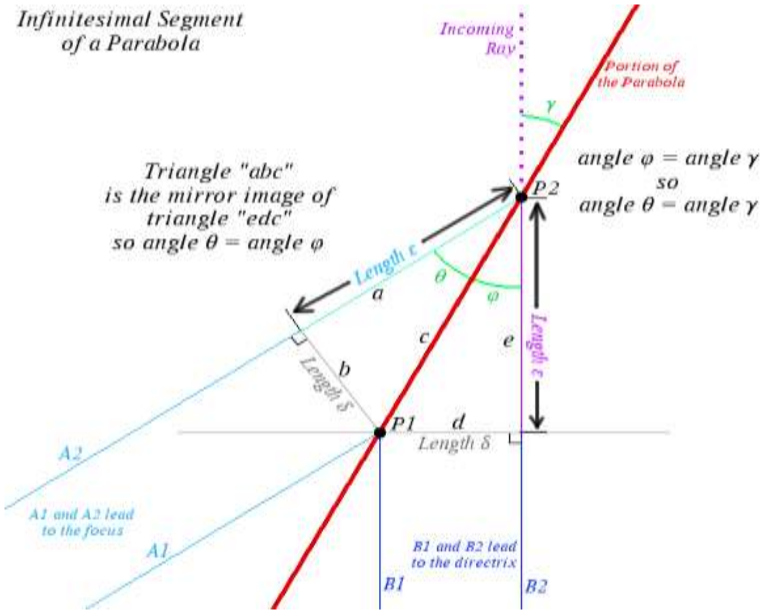


Fig. 3

Lines $A1$ and $B1$ lead from point p_1 to the focus and directrix, respectively. Since p_1 is on the parabola, lines $A1$ and $B1$ must be the same length. Just a little farther along the parabola we have marked point p_2. We've drawn line $A2$ from p_2 to the focus, and we've drawn line $B2$ straight down to the directrix. As mentioned, p_1 and p_2 are actually so close together that $A1$ and $A2$ are essentially parallel. Segment b runs perpendicularly from the end of $A1$ to $A2$. $A2$ is longer than $A1$ by the piece extending past segment b, marked "a"; it is ε units long. Segment d runs perpendicularly from the end of $B1$ to $B2$, and e is the "extension" of $B2$ versus $B1$. Since p_2 must also be equidistant from the directrix and the focus, segment e must also be ε units long. That is the key to the proof! Triangles abc and edc are right triangles and two of their sides (a and e, and the hypotenuse, which is c for each of them) are certainly the same length. So, the third pair of sides, b and d, must also be the same length. So, if lines $A1$ and $A2$ are δ units apart, then $B2$ and $B2$ must also be δ units apart.

So, triangles abc and edc are just mirror images, so angles θ and φ must be identical. Angles φ and γ are also identical, since they're opposite angles of two intersecting lines. But then angles θ and γ must also be identical. A light ray coming straight down from the top of the page (parallel to the parabola's axis), along line $B2$, strikes the parabola at angle γ. It's naturally reflected at the same angle so, since $\gamma = \theta$, the ray will head down line $A2$, straight to the focus, as was to be shown [3].

The reflection of light in parabola

The phenomenon of incident rays at a point on a surface where total reflection occurs has been extensively studied by Fermat's principle, light always travels along the path that takes the least amount of time [6]. According to the principles of optics, the incident ray makes an angle on the line perpendicular to the surface equal to the angle that the reflected ray makes on that perpendicular. Furthermore, the line perpendicular to the surface that passes through the point of incidence, the incident ray at that point, and the reflected ray are in the same plane. In the experiment that is proposed, the ray falls on a surface that is not flat, so at that point the tangent plane to the surface is considered. The surface with parameters $s, t \in R$ has equations[3]:

$$\begin{cases} x = \frac{1}{2p} s^2 \\ y = s \\ z = t \end{cases}$$

Optical property of a parabola

● rays leaving the focus of the parabola, reflected from it, will go parallel axes of symmetry;

● rays that arrived parallel to the axis of symmetry of the parabola, reflected from it, will come into focus.

Let us formulate the mentioned optical property of a parabola. If a point source of light (a light bulb) is placed at the focus of the parabola and turned on, then the rays, reflected from the parabola, will go parallel to the symmetry axis of the parabola, and the leading front will be perpendicular to the axis fig. 4.

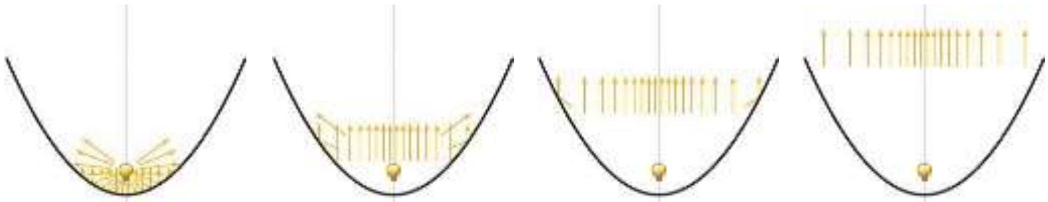


Fig. 4

The opposite is also true if a stream of rays' parallel to the axis of symmetry falls on a parabola, then, reflected from the parabola, the rays will come into focus; and they will arrive simultaneously if the leading front of the ray flow is perpendicular to the axis fig. 5.

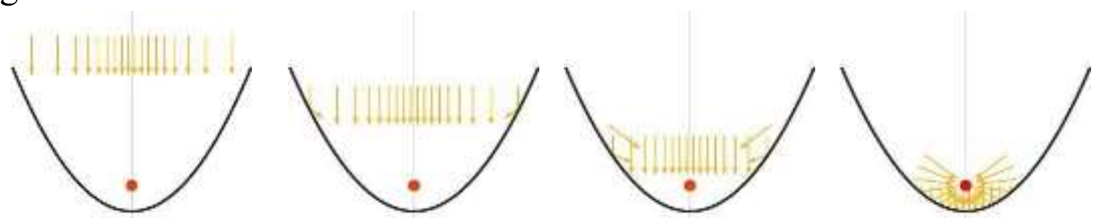


Fig. 5

The parabolic billiards model demonstrates the optical property of a parabola using mechanics fig. 6 [2].

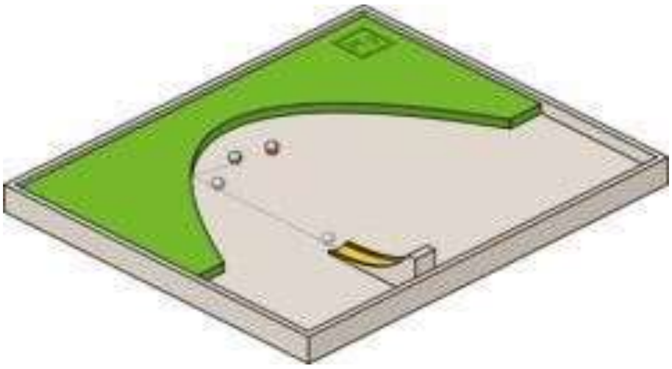


Fig. 6

Isaac Newton noticed that when a cylindrical vessel rotates, the surface liquid poured into it takes the shape of a paraboloid, and explained this phenomenon using laws he himself found. Nowadays, this effect is used in the manufacture large parabolic mirrors for telescopes - this method is faster and cheaper than classic grinding. And sometimes "temporary" telescopes with a liquid mirror are created: The vessel with mercury is rotated only during observations fig. 7 [3].

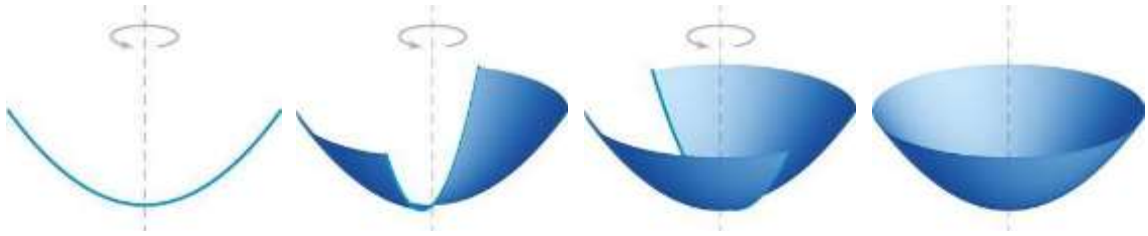


Fig. 7

Cross sections of television antenna dishes are parabolic in shape. Use the fig. 8 shown to write a paragraph explaining why these dishes are parabolic [1,7].

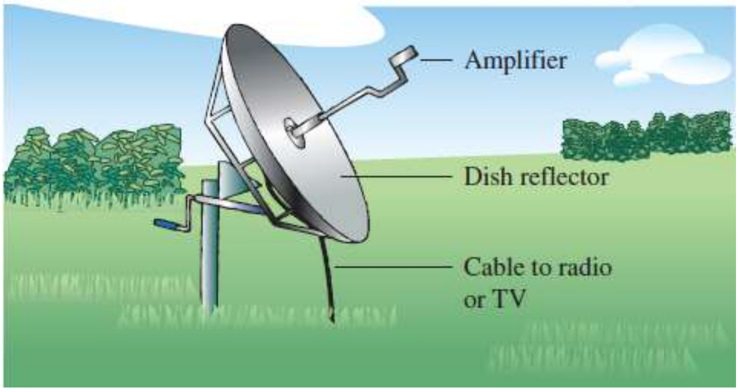


Fig. 8

Experiment

To evaluate the effectiveness of using a parabolic heating plate water, we will conduct the following experiment and solve the problem of finding the absorbed the amount of heat in a body at focus fig. 9. Problem: if, install an iron parabolic plate and direct it concave side towards the Sun while placing a vessel with water in focus, the water will begin to heat up. Determine the maximum temperature of the liquid and the amount of heat absorbed. To conduct the experiment, we needed the following equipment: iron parabolic plate, tripod, beaker with water, electronic thermometer, stopwatch. Knowing that the amount of heat is the energy that a body receives or gives off in heat transfer process. Denoted by the symbol Q, it is measured, like any energy, in Joules. As a result of various heat exchange processes, the energy that is transferred is determined in my own way. Since in our problem the process is heating and this process is characterized change in body temperature. The amount of heat is determined by the formula:

$$\mathrm{Q} = \mathrm{mc}(\mathrm{t}_2 - \mathrm{t}_1) = \mathrm{mc}\Delta\mathrm{t}$$

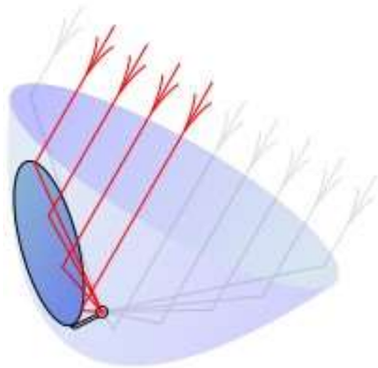


Fig. 9

After conducting a series of experiments, we compiled a table of the results obtained tab. 1 and also compiled a graph from tab. 2 that shows. That water weighing 5 grams received the most heat in 20 minutes when conducting an experiment indoors on a sunny day. Having conducted an experiment under the same weather conditions but outside, we see that heating water is less effective, since there are external factors that do not allow us to achieve the same results. Next, we decided to conduct an experiment and find out whether we could heat water to the same temperature from an incandescent lamp by placing a parabolic plate at a distance of 70 cm. As a result, the maximum temperature in 20 minutes could reach 37.5°C.

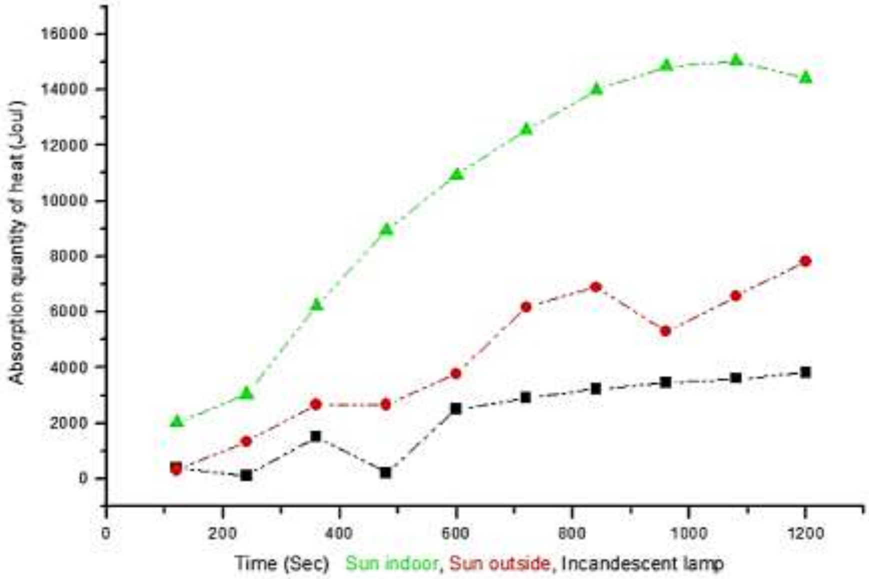
Tab. 1

Light of source	Heated body	Specific heat capacity $C, \frac{J}{Kg,ses}$	Body mass	Temperature t_1, C^0	Temperature t_2, C^0	Measure of temperature $\Delta t, C^0$	Absorption quantity of heat Q,J	Time t
Sun outside	Water	4200	0,005	21	62,1	41,1	863,1	1320
Sun indoors	Water	4200	0,005	17,4	80,7	63,3	1329,3	1320
Incandescent lamp	Water	42200	0,005	18,5	37,5	19	399	1320

Tab. 2

Time (***Sec***)	***Sun indoor***	***Sun outside***	***Incandescent lamp***
120	41,1	31,9	202,6
240	105	134,4	304,5
360	150,6	266,7	623,7
480	210	266,7	894,6
600	250,1	378	1094,1

720	290,9	617,4	1255,8
840	323,4	688,8	1398,6
960	346,5	529,2	1484,7
1080	360,6	657,3	1504,1
1200	380,5	781,2	1442,7
1320	399	863,1	1329,3



Discussion: The elements of the parabola in space can be determined analytically by applying an abatement of the plane using matrix techniques for calculating eigenvalues, applying rotations and translations. Practical experiments to verify the laws of total reflection can be proposed together with developments with applications such as GeoGebra and Matlab, interconnecting different mathematical topics. The establishment of intra-curricular connections with extra mathematical knowledge is in line with the new teaching paradigm where the use of ICT tools provides added value, promoting the development of thought. It is conjectured that the project can be accepted by other high school groups, given the good results that have been obtained in the experience referred to in the previous section. Other models of structures could be set up to further investigate the incident and reflected ray equations, checking the effects when the surface is changed. using a parabolic dish as a heater requires detailed modification of the parabolic plate. For example, pasting the surface with small pieces of mirror. This will allow the sun's rays to be better reflected from the surface of the plate. This is what we plan to do in the near future. The principle of focusing the sun's rays used in inexpensive culinary solar ovens and barbecues, as well as for solar pasteurization of water. The use of parabolic dishes will reduce the negative environmental impact. The main goal of the work has been achieved.

Conclusion

The parabola is an effective tool in the hands of an engineer; it can be used to solve a wide range of technical problems in various devices and instruments. In addition, a parabola, as a conic section, is already incorporated into the operating principle of many technologies in a cone lantern light source. A parabola is just a geometric curve, but has many applications due to its unusual properties.

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