Funksional qatorlar

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Annotatsiya: Ushbu ishda funksional qatorlar nazariyasining asosiy tushunchalari va muhim xossalari tahlil qilinadi. Funksional qator tushunchasi, uning yaqinlashish masalalari hamda nuqtaviy va bir xilda yaqinlashish tushunchalari batafsil o‘rganiladi. Qatorlarning yaqinlashishini tekshirishda qo‘llaniladigan Veyershtrass mezoni, Dirixle va Abel mezonlari keltirilib, ularning amaliy qo‘llanishi misollar orqali yoritiladi.

Kalit so‘zlar: qator yaqinlashuvchi, uzoqlashuvchi, funksional qatorlar

Functional series

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Abstract: This work analyzes the basic concepts and important properties of the theory of functional series. The concept of a functional series, its convergence issues, and the concepts of pointwise and uniform convergence are studied in detail. The Weierstrass criterion, Dirichlet and Abel criteria used to check the convergence of series are presented, and their practical application is illustrated by examples.

Keywords: series converging, receding, functional series

Agar qatorning har bir hadi biror E sohada aniqlangan $z = x + iy$ o‘zgaruvchilarning bir qiymatli $f_n(z)$ funksiyasidan iborat bo‘lsa, ya’ni $f_,(z) + f_2(z) + ... + f_n(z) + ... = \sum_{n=1}^{\infty} f_n(z)$ (1) ga funksional qator deyiladi.

Agar E ga tegishli biror z_0 o‘zgarmas kompleks sonni (1) ga qo‘ysak sonli $f_1(z_0) + f_2(z_0) + ... + f_n(z_0) + ... = \sum_{n=1}^{\infty} f_n(z_0)$ (2) sonli qator hosil bo‘ladi.

Agar (2) yaqinlashuvchi bo‘lsa, $\alpha_1 + \alpha_2 + ... + \alpha_n + ... = \sum_{n=1}^{\infty} \alpha_n$ qator z_0 nuqtada yaqinlashuvchi bo‘ladi. (2) qatorning yaqinlashuvchi nuqtalarning to‘plami (1) ning yaqinlashuvchi sohasi deyiladi. Agar (1) qator biror E sohaning barcha nuqtalarida yaqinlashuvchi bo‘lsa, u E sohada yaqinlashuvchi va uning yig‘indisi ham E ga tegishli biror $z = x + iy$ nuqtadagi aniq bir $f(z)$ funksiyadan iborat bo‘ladi, ya’ni $f_1(z) + f_2(z) + ... + f_n(z) + ... = \sum_{n=1}^{\infty} f_n(z)$ (1)

Agar $s_n(z) = f_1(z) + ... + f_n(z)$ deb belgilasak, $R_n(z) = f(z) - s_n(z) =$ = $f_{n+1}(z) + f_{n+2}(z) + \cdots$ (3) ga (1) qatorning qoldiq hadi deyiladi.

Teorema (Veyershtrass teoremasi). Agar E sohada $\alpha_1 + \alpha_2 + ... + \alpha_n + ... = \sum_{n=1}^{\infty} \alpha_n$ qatorning barcha hadlarning moduli $\sum_{n=1}^{\infty} a_n$ musbat hadli yaqinlashuvchi sonli qator hadlaridan katta bo‘lmasa, u holda $\alpha_1 + \alpha_2 + ... + \alpha_n + ... = \sum_{n=1}^{\infty} \alpha_n$ qator tekis yaqinlashuvchi bo‘ladi. $|f(z_n)| < a_n$

Teorema. Agar E da barcha hadlari uzliksiz bo‘lgan $\alpha_1 + \alpha_2 + ... + \alpha_n + ... = \sum_{n=1}^{\infty} \alpha_n$ qator tekis yaqinlashuvchi bo‘lsa, u qatorning yig‘indisi $f(z) = \sum_{n=1}^{\infty} f_n(z)$ ham E sohada uzluksiz bo‘ladi.

Darajali qatorlar

Funksional qatorlarning xususiy ko‘rinishi bo‘lgan darajali qatorlar amalyotda ko‘proq ishlatiladi.

Ushbu ko‘rinishdagi. $C_0 + C_1(z-a) + C_2(z-a)^2 + ... + C_n(z-a)^n + ...$ (4) yoki $C_0 + C_1 z + C_2 z^2 + ... + C_n z^n + ...$ (5) qatorlar darajali qatorlar deyiladi. Bunda $a = \alpha + i\beta$, $C_n = a_n + ib_n$ kompleks sonlardir.

Teorema. (Abel teoremasi). Agar (4) qator biror nuqtada yaqinlashuvchi bo‘lsa, bu qator $|z-a| < |z_0 - a|$ doiraning hamma ichki nuqtalarida absolyut yaqinlashadi (1-chizma).

Natija. Agaar (4) qator biror $z = z_0$ nuqtada uzoqlashuvchi bo‘lsa $|z-a| < |z_0 - a|$ tegsizlikni qanoatlantiruvchi ixtiyoriy nuqtada uzoqlashuvchi bo‘ladi.

Misol. $\sum_{n=1}^{\infty} (\frac{z}{n})^n$ qator ixtiyoriy nuqtada absolyut yaqinlashuvchi, chunki z har qanday bo‘lsa ham biror n katta raqamdan boshlab $\frac{|z|}{n} < \frac{1}{2}$ bo‘ladi, yoki $\left|\frac{z^n}{n^n}\right| = \left|\frac{|z|}{n}\right|^n < \frac{1}{2}$

, $\sum_{n=1}^{\infty}\frac{1}{2^n}$ qator yaqinlashuvchi. Umuman (4) yoki (5) ko‘rinishidagi darajali qatorlaring yaqinlashish doirasining radiusi Dalamder – Adamar formulasi yordamida topiladi.

1. Koshi formulasi: $R=\frac{1}{L}=\frac{1}{\lim\limits_{n\to\infty}\sqrt[n]{C_n}}$ (1)

2. Dalamber formulasi: $R=\frac{1}{L}=\lim\limits_{n\to\infty}\left|\frac{c_n}{c_{n+1}}\right|$ (2)

3. Koshi-Adamar forulasi: $R=\frac{1}{\lim\limits_{n\to\infty}\sqrt[n]{|C_n|}}$ (3) bo‘lsa, $|z|<R$ yaqinlashish doirasi bo‘ladi.

Misol. $\sum_{n=1}^{\infty}\left(\frac{z}{in}\right)^n$ qatorining yaqinlashish radiusini toping.

Yechish. $C_n=\frac{1}{(in)^n}, L=\lim\limits_{n\to\infty}\sqrt[n]{c_n}=\lim\limits_{n\to\infty}\sqrt[n]{\frac{1}{(in)^n}}=\lim\limits_{n\to\infty}\frac{1}{in}=0.$

Demak, qatorning yaqinlashish radiusi (sohasi): $R=\frac{1}{L}=\frac{1}{0}=\infty$ bo‘lgani uchun yaqinlashish sohasi butun kompleks tekislikdan iborat.

Misol. $\sum_{n=1}^{\infty}e^{in}z^n$ qatorini yaqinlashish sohasini toping.

Yechish. $C_n=e^{in}$, $L=\lim\limits_{n\to\infty}\sqrt[n]{|e^{in}|}=\lim\limits_{n\to\infty}|e^i|=e^i=1,$ $R=|e^i|=e^i=1$, $R=\frac{1}{L}=1$, $|z|<1$ bo‘lgani uchun yaqinlashish sohasi birlik doiraning ichki nuqtalaridan iborat.

Foydalanilgan adabiyotlar

1. S.X. Sirojiddinov, M.Maqsudov, M.S.Salohiddinov. Kompeleks o’zgaruvchining funksiyalari nazariyasi-T,: O’qituvchi, 1979

2. Sh. T. Maqsudov. Analitik funksiyalar nazariyasidan mashqlar-T.: O’qituvchi,

3. I. I.Privalov. Vvedenie v teoriyu funksiy kompleksnogo peremennogo.-M.:

4. A.I. Markushevich. Kratkiy kurs teorii analiticheskix funksiy-M Fizmatgiz

5. Ya. S. Bugrov, S.M.Nikolskiy. Funksii Komleksnogo peremennogo-M,: Nauka