Bir hosil qilingan operatorning qo‘zg‘almas nuqtalari haqida

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Annotatsiya: Matitsasi ustun bo‘yicha stoxastik bo‘lgan chiqli operator va bitta Volterra tipidagi operatorning qo‘zg‘almas nuqtalari o‘rganilgan.

Kalit so‘zlar: chiqli operator, kvadratik stoxastik operator, superpozitsiya, qo‘zg‘amas nuqta

On the fixed points of an induced operator

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Abstract: Fixed points of a linear operator with a column-stochastic matrix representation, as well as those of a Volterra-type operator, are investigated.

Keywords: linear operators, quadratic stochastic operators, superposition, fixed points

Ixtiyoriy, jumladan kvadrarik stoxastik operatorlarning qo’zg’almas nuqtalarini va tadbiqlarini o’rganish muhim masalalardan biri masalardan hisoblanadi.

Kvadratik stoxastik operatorlar va ularning tadbiqlari bo’yicha juda ko’p izlanishlar olib borilgan va hozirda ham olib borilmoqda [1-20].

Bizga X va Y chiziqli normallangan fazolar berilgan bo‘lsin.

1-ta’rif. X fazodan olingan har bir x elementga Y fazoning yagona y elementini mos qo‘yuvchi:

$$Ax = y \quad (x \in X, y \in Y)$$

akslantirish operator deyiladi.

Umuman A operator X ning barcha nuqtalarida aniqlangan bo‘lishi shart emas. Bu holda Ax mavjud va Ax ∈ Y bo‘lgan barcha x ∈ X lar to‘plami A operatorning aniqlanish sohasi deyiladi va D(A) bilan belgilanadi, ya’ni:

$$D(A) = \{x \in X : Ax \text{ mavjud va } Ax \in Y\}$$

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Odatda D(A) ning chiziqli yopiq bo‘lishi (26.6-ta’rifga qarang) talab qilinadi. Ya’ni, agar x, y ∈ D(A) bo‘lsa, u holda ixtiyoriy α, β ∈ ℂ lar uchun αx + βy ∈ D(A) bo‘ladi.

2-ta’rif. Agar ixtiyoriy x, y ∈ D(A) elementlar va ixtiyoriy α, β ∈ ℂ sonlar uchun

$$\alpha x+\beta y \in D(A)$$

bo‘lib,

$$A(\alpha x + \beta y) = \alpha Ax + \beta Ay$$

tenglik o‘rinli bo‘lsa, A ga chiziqli operator deyiladi.

3-ta’rif. A va B chiziqli operatorlarning *yigindisi* deb A+B kabi belgilanadigan va (A+B)x=Ax+Bx tenglik bilan aniqlanadigan yangi bir operatorga aytiladi.

4-ta’rif. A va B operatorlarning *ko‘paytmasi* deb A·B kabi belgilanadigan va (A·B)(x)=A(B(x)) tenglik bilan aniqlanadigan operatorga aytiladi.

Chiziqli bo`lmagan dinamik tizimlar, jumladan kvadratik va kubik stoxastik operatorlar matematik genetika, fizika va kimyo sohasiga doir masalarning matematik modelini ifodalaydi. Kvadratik va kubik stoxastik operator yo`nalishida qator ilmiy izlanishlar olib borilgan.

Stoxastiklik (qadimgi yunoncha tos so‘zdan olingan bo‘lib, maqsad, taxmin) - tasodifiylikni anglatadi. Tasodifiy (stoxastik) jarayon deterministik bo‘lmagan jarayon bo‘lib, bunday sistemaning keyingi holati ham taxmin qilinadigan, ham tasodifiy miqdorlar bilan tavsiflanadi.

Xususan, biologiyada populyatsiya evolyusiyasining matematik modeli kvadratik stoxastik operatorlar orqali ifodalanadi. Ushbu talqin bilan yopiq biologik sistema evolyusiyasi jarayonida har xil tipdagi individlarning chegaralangan taqsimotini topish muammosi kvadratik stoxastik operatorning asimptotik xususiyatlarini o‘rganilishiga tengdir. Bundan tashqari, kvadratik stoxastik operatorlar nazariyasida oddiy va nostandart masalalar hamda yechilmagan masalalarning ko‘pligi matematik nuqtai nazardan katta qiziqish uyg‘otadi.

5-ta’rif. R^n da

$$S^{n-1} = \left\{ x = (x_1,..., x_n) : \sum_{i=1}^{n} x_i = 1, x_i \geq 0 \right\}$$

nuqtalar to`plamiga $(n-1)$ o`lchovli simpleks deyiladi.

$V : S^{n-1} \to S^{n-1}$ kvadratik stoxastik operator quyidagicha aniqlanadi:

$$(Vx)_k = x_k^{'} = \sum_{i,j=1}^{n} P_{ij,k} x_i x_j, k = \overline{1,n}$$

bu yerda

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$$P_{ij,k} \geq 0,\ P_{ij,k} = P_{ji,k},\ \sum_{i,j=1}^{n} P_{ij,k} = 1,\ x = (x_1,\ldots,\ x_n) \in S^{n-1}.$$

6-ta'rif. Kvadratik stoxastik operator $V : S^{n-1} \to S^{n-1}$ Volterra tipidagi deyiladi, agar

$$P_{ij,k} = 0,\ \ k \notin \{i, j\}$$

quyidagi tasdiqlar o`rinli bo`lsa.

7-ta'rif. Faraz qilaylik, A va B - ikkita chiziqli operator bo'lib, ulardan A E fazodan E_1 fazoga B esa E_1 fazodan E_2 fazoga amal qiladi. x$\in E$ elementga E_2 fazoning z=B(Ax) elementini mos qo'yuvchi operatorga A va B operatorlarning ko'paytmasi (superpozitsiyasi) deb ataladi.

8-ta'rif. Agar V(x) = x tenglik bajarilsa,x $\in S^{n-1}$ nuqta kvadratik stoxastik operator V uchun qo'zg'almas nuqta deyiladi.

Biz A operator bilan aniqlangan chiziqli va V operator bilan aniqlangan kvadratik stoxostik operatorlarning superpozisiyasini qaraymiz:

$$A = \begin{pmatrix} 1-\alpha-\beta & 1-\gamma & 1 \\ \beta & \gamma & 0 \\ \alpha & 0 & 0 \end{pmatrix},$$

$$V_{a,b,c}(x) = \begin{cases} x_1' = x_1(1 + ax_2 - bx_3), \\ x_2' = x_2(1 - ax_1 + cx_3), \\ x_3' = x_3(1 + bx_1 - cx_2). \end{cases}$$

$$B = A\left(V_{a,b,c}(x)\right) = \begin{pmatrix} 1-\alpha-\beta & 1-\gamma & 1 \\ \beta & \gamma & 0 \\ \alpha & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1(1 + ax_2 - bx_3) \\ x_2(1 - ax_1 + cx_3) \\ x_3(1 + bx_1 - cx_2) \end{pmatrix} =$$

$$= \begin{pmatrix} \Lambda \\ \beta x_1(1 + ax_2 - bx_3) + \gamma x_2(1 - ax_1 + cx_3) \\ ax_1(1 + ax_2 - bx_3) \end{pmatrix},$$

bu yerda

$$\Lambda = (1-\alpha-\beta)x_1(1 + ax_2 - bx_3) + (1-\gamma)x_2(1 - ax_1 + cx_3) + \\ + x_3(1 + bx_1 - cx_2).$$

Demak

$$\begin{cases} x_3 = \Lambda, \\ x_2 = \beta x_1(1 + ax_2 - bx_3) + \gamma x_2(1 - ax_1 + cx_3) \\ x_1 = \alpha x_1(1 + ax_2 - bx_3) \end{cases} (1)$$

B(B(x)=x) operatori uchun qo'zg'almas nuqtalarni topamiz. Shunda (1) sistema quyidagi ko'rinishga ega bo'ladi:

$$\begin{cases} x_1 = \Lambda_1, \\ x_2 = \beta x_1(1 + ax_2 - bx_3) + \gamma x_2(1 - ax_1 + cx_3) \\ x_3 = \alpha x_1(1 + ax_2 - bx_3) \end{cases} (2)$$

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$$\Lambda_1 = (1-\alpha-\beta)x_1(1+ax_2-bx_3)+(1-\gamma)x_2(1-ax_1+cx_3)+ \\ +x_3(1+bx_1-cx_2).$$

1. $\alpha=\gamma=1$ bo'lsin. Bu holda (2) sistema quyidagi ko'rinishga kelib qoladi

$$\begin{cases} x_1 = x_3(1+bx_1-cx_2) \\ x_2 = x_2(1-ax_1+cx_3) \\ x_3 = x_1(1+ax_2-bx_3) \end{cases} \quad (3)$$

(3) sistemaning 2-tengligidan quyidagi kelib chiqadi:

$$(1-1+ax_1-cx_3)x_2=0, (ax_1-cx_3)x_2=0.$$

a) $ax_1-cx_3=0, ax_1=cx_3\ , x_1=\frac{c}{a}x_3$

(3) sistemaning 1-tengligidan quyidagi kelib chiqadi

$$\frac{c}{a}x_3 = x_3(1+bx_1-cx_3), \frac{c}{a}=1+x_3\left(\frac{bc}{a}-c\right)$$

$$x_3=\frac{\frac{c}{a}-1}{c\left(\frac{b}{a}-1\right)}=\frac{c-a}{c(b-a)}, x_1=\frac{c}{a}\frac{c-a}{c(b-a)}=\frac{c-a}{a(b-a)}$$

$$x_2=1-x_1-x_3=1-\frac{c-a}{a(b-a)}-\frac{c-a}{c(b-a)}= \\ =\frac{a\,c(b-a)-c+a-c+a}{a\,c(b-a)}\ ;$$

$$M_1=\left(\frac{c-a}{a(b-a)}\ ;\ \frac{a\,b\,(c-2)-a(ac-2)}{a\,c\,(b-a)}\ ;\ \frac{c-a}{c(b-a)}\right)-$$

qo' zg almas nuqta.

b) $x_2=0$ bo'lsa, (3) sistemaning 3-tengligidan quyidagi kelib chiqadi

$$x_3=x_1(1-bx_3), x_3=x_1-bx_1x_3, x_3(1+bx_1)=x_1,$$

$$x_3=\frac{x_1}{1+bx_1},$$

$$x_1+\frac{x_1}{1+bx_1}=1,\ x_1\left(\frac{1}{1+bx_1}\right)=1, x_1\frac{2+bx_1}{1+bx_1}=1$$

$$2x_1+bx_1=1+bx_1, bx_1^2+(2-b)x_1-1=0$$

$$x_{1_{(1,2)}}=\frac{b-2\pm\sqrt{(2-b)^2+4b}}{2b}=\frac{b-2\pm\sqrt{4-4b+b^2+4b}}{2b}= \\ =\frac{b-2\pm\sqrt{4+b^2}}{2b};\ x_1=\frac{b-2+\sqrt{4+b^2}}{2b},$$

$$x_3=\frac{\frac{b-2+\sqrt{4+b^2}}{2b}}{1+b\frac{b-2+\sqrt{4+b^2}}{2b}}=\frac{b-2+\sqrt{4+b^2}}{2b+b\left(b-2+\sqrt{4+b^2}\right)}=\frac{b-2+\sqrt{4+b^2}}{b^2+b\sqrt{4+b^2}},$$

$$M_2 = \left(\frac{b-2+\sqrt{4+b^2}}{2b}, 0, \frac{b-2+\sqrt{4+b^2}}{b^2+b\sqrt{4+b^2}}\right) -$$

fixed point in this case

1-teorema. $\alpha = \gamma = 1$ bo'lganda B operator M_1 va M_2 nuqtalarga ega.

2. $\alpha = 1, 0 < \gamma < 1$ bo'lsin. Bu holda (2) sistemadan quyidagi ko'rinishni oladi:

$$\begin{cases} x_1 = (1-\gamma)x_2(1-ax_1+cx_3) + x_3(1+bx_1-cx_2) \\ x_2 = \gamma x_2(1-ax_1+cx_3) \\ x_3 = x_1(1+ax_2-bx_3) \end{cases} \quad (4)$$

(4) sistemaning 2-tengligidan

$$x_2 - \gamma x_2(1-ax_1+cx_3) = 0$$

$$\big(1-\gamma(1-ax_1+cx_3)\big)x_2 = 0;$$

$$a)\ 1-\gamma(1-ax_1+cx_3) = 0, 1-\gamma+\gamma(ax_1-cx_3) = 0$$

$$ax_1 - cx_3 = \frac{\gamma-1}{\gamma}\ 1-\frac{1}{\gamma} < 0, ax_1 = \frac{\gamma-1}{\gamma} + cx_3$$

$$x_1 = \frac{\frac{\gamma-1}{\gamma}+cx_3}{a};$$

(4) sistemaning 3-tengligidan

$$x_3 = \frac{\frac{\gamma-1}{\gamma}+cx_3}{a}(1+ax_2-bx_3),$$

$$ax_3 = \left(\frac{\gamma-1}{\gamma}+cx_3\right)(1+ax_2-bx_3),$$

$$ax_3 = \left(\frac{\gamma-1}{\gamma}+cx_3\right)(1-bx_3) + \left(\frac{\gamma-1}{\gamma}+cx_3\right)ax_2\ ;$$

$$ax_3 - \left(\frac{\gamma-1}{\gamma}+cx_3\right)(1-bx_3) = \left(\frac{\gamma-1}{\gamma}+cx_3\right)ax_2,$$

$$x_2 = \frac{ax_3 - \left(\frac{\gamma-1}{\gamma}+cx_3\right)(1-bx_3)}{a\left(\frac{\gamma-1}{\gamma}+cx_3\right)} +$$

$$+\frac{\frac{\gamma-1}{\gamma}+cx_3}{a} + \frac{ax_3 - \left(\frac{\gamma-1}{\gamma}+cx_3\right)(1-bx_3)}{a\left(\frac{\gamma-1}{\gamma}+cx_3\right)} + x_3 = 1,$$

$$\left(\frac{\gamma-1}{\gamma}+cx_3\right)^2 + ax_3 - \left(\frac{\gamma-1}{\gamma}+cx_3\right)(1-bx_3) +$$

$$+(x_3-1)a\left(\frac{\gamma-1}{\gamma}+cx_3\right) = 0,$$

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$$\left(\frac{\gamma-1}{\gamma}\right)^2+2\left(\frac{\gamma-1}{\gamma}\right)cx_3+c^2x_3^2+ax_3-$$
$$-\left(\frac{\gamma-1}{\gamma}\right)+\left(\frac{\gamma-1}{\gamma}\right)bx_3-cx_3+bcx_3^2+$$
$$+a\left(\frac{\gamma-1}{\gamma}\right)x_3+acx_3^2-a\left(\frac{\gamma-1}{\gamma}\right)-acx_3=0;$$
$$c^2x_3^2+bcx_3^2+acx_3^2+$$
$$+\left(2\frac{(\gamma-1)c}{\gamma}+a+\frac{(\gamma-1)b}{\gamma}-c+\frac{(\gamma-1)a}{\gamma}-ac\right)x_3+$$
$$+\left(\frac{\gamma-1}{\gamma}\right)^2-\left(\frac{\gamma-1}{\gamma}\right)-a\left(\frac{\gamma-1}{\gamma}\right)=0,$$
$$a(a+b+c)x_3^2+$$
$$+\left(c\left(2\gamma-\frac{3}{\gamma}-a\right)+\frac{\gamma(a+b)}{\gamma}+(a-c)\sqrt{x_3}+\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\gamma-1-1-a}{\gamma}\right)\right)=0.$$

$$x_{3_{(1,2)}}==\frac{-[c(2\gamma-3-a)+\gamma(a+b)-b]\pm\sqrt{\Delta}}{2c(a+b+c)},$$

$$x_{3_{(1,2)}}^{*}=\frac{-\left[[c(2\gamma-3-a)+\gamma(a+b)-b]\pm\sqrt{\Delta}\right]}{2c(a+b+c)},$$

$$\Delta=\left[[c(2\gamma-3-a)+\gamma(a+b)-b]\right]^2-4c(a+b+c)(\gamma-1)(\gamma-2-a),$$

$$x_1^*=\frac{\gamma-1+cx_3^*}{a},$$

$$x_2^*=\frac{ax_3^*-(\gamma-1+cx_3)(1-bx_3)}{a(\gamma-1+cx_3)},$$

$$Fix(B)=M_3=(x_1^*,x_2^*,x_3^*).$$

b) $x_2=0 \Rightarrow\ x_3=x_1(1-bx_3), x_3=(1-x_3)(1-bx_3),$

$$x_3=1-bx_3-x_3+bx_3{}^2;\ bx_3{}^2-(2+b)x_3+1=0;$$

$$x_{3_{(1,2)}}=\frac{2+b\pm\sqrt{(2-b)^2-4b}}{2b}=\frac{2+b\pm\sqrt{4+b^2}}{2b};$$

$$x_3=\frac{2+b-\sqrt{4+b^2}}{2b},$$

$$x_1=1-\frac{2+b-\sqrt{4+b^2}}{2b}=\frac{2b-2-b+\sqrt{4+b^2}}{2b}=\frac{b-2+\sqrt{4+b^2}}{2b};$$

$$Fix(B)=\left(\frac{b-2+\sqrt{4+b^2}}{2b},0,\frac{2+b-\sqrt{4+b^2}}{2b}\right)$$

2-teorema. $\alpha=1, 0<\gamma<1$ bo'lganda B operator M_3 va M_4 qo'zg'almas nuqtalarga ega.

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