# Uch noma'lumli nochiziqli ayrim oddiy differensial tenglamalar sistemalarining analitik yechimlarini qurish va ularning yaqinlashishini tahlil qilish

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**Annotatsiya:** Ushbu maqolada uch noma'lumli nochiziqli ayrim oddiy differensial tenglamalar sistemalari uchun Koshi masalasining analitik yechimlarni qurish o'rganilgan. Yechimning mavjudligi, yagonaligi va analitikligiga oid shartlar aniqlangan. Vaqt bo'yicha qator koeffitsientlari uchun rekurrent formula, lokal yaqinlashuv radiusi va qoldiq had bahosi olingan. Uch no'malumli kvadratik sistema misolida kichik parametr bo'yicha ikkita yechim qurilgan va parametrik qatorning barcha musbat vaqtlarda tekis yaqinlashishi uchun etarli shart isbotlangan. SHu bilan bir qatorda, Volterra sistemalari umumiy usulning xususiy tatbiqi sifatida qaralgan.

**Kalit so'zlar:** nochiziqli oddiy differensial tenglamalar sistemasi, Koshi masalasi, analitik yechim, darajali qator, rekurrent formula, kichik parametr, majoranta, Volterra tipidagi sistema

# Construction of analytic solutions to certain three-variable nonlinear systems of ordinary differential equations and analysis of their convergence

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**Abstract:** This paper investigates the construction of analytic solutions to the Cauchy problem for certain nonlinear systems of ordinary differential equations in three unknowns. Conditions ensuring the existence, uniqueness, and analyticity of solutions are established. A recurrence formula for the coefficients of the power series in time, a local radius of convergence, and an estimate of the remainder term are obtained. For a three-variable quadratic system, two solutions are constructed using a small-parameter expansion, and a sufficient condition for the uniform convergence of the parameter series for all positive times is proved. In addition, Volterra-type systems are considered as a particular application of the general method.


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**Keywords:** nonlinear systems of ordinary differential equations, Cauchy problem, analytic solution, power series, recurrence formula, small parameter, majorant, Volterra-type system

Kirish. Uch noma'lumli nochiziqli ayrim oddiy differensial tenglamalar sistemalarida o'zgaruvchilarning o'zaro ta'siri analitik ko'rinishdagi yechimni olishga to'sqinlik qiladi. Shu sababli yechimni analitik qatorga yoyish, koeffitsientlarini rekurrent aniqlash va yaqinlashuvini asoslash orqali analitik yechimlar izlanadi.

Koshi masalasining klassik asoslari [1], analitik differensial tenglamalarning kompleks usullari [2], kichik parametr metodologiyasi [3] ga tayanib, ushbu ishda umumiy rekurrent sxema va hisoblanadigan yaqinlashuv baholari chiqarilgan.

Ikki xil qator farqlanadi: umumiy sistema uchun lokal vaqt qatori va maxsus sistema uchun global parametrik qator.

Koshi masalasi va regulyarlik shartlari. Quyidagi umumiy uch noma'lumli sistemani ko'ramiz:

$$\dot{u}_i(t) = F_i\big(t, u_1(t), u_2(t), u_3(t)\big), \quad i = 1,2,3.$$
$$u_i(t_0) = a_i, \quad i = 1,2,3; \qquad u = (u_1, u_2, u_3)^{\mathsf{T}}.$$

Bu erda $F = (F_1, F_2, F_3)^{\mathsf{T}}$ kvadratik shakl bilan cheklanmaydi: masalan, $u_1^3$, $\sin u_3$, $e^{u_1 u_2}$ kabi hadlarga ham yo'l qo'yiladi.

Asosiy qism. Agar $F$ boshlang'ich nuqta atrofida uzluksiz bo'lsa, lokal yechim mavjudligi haqida klassik natija qo'llanadi; bu shartning o'zi yagonalikni ta'minlamaydi. $F$ holat o'zgaruvchilari bo'yicha lokal Lipshits shartini bajarsa, yagonalik kelib chiqadi. Masalan, tegishli to'plamda

$$\|F(t,v) - F(t,w)\|_\infty \le L\|v - w\|_\infty$$

bo'lishi etarli. Bu erda $\|v\|_\infty = \max_i |v_i|$. $F$ haqiqiy analitik bo'lsa, kompleks davom orqali analitik yechim quriladi [4]. Uzluksizlik yoki silliqlikning o'zi analitiklikni kafolatlamaydi.

Har bir klassik yechim uchun sistema integral shaklga keladi:

$$u(t) = a + \int_{t_0}^{t} F\left(s, u(s)\right) ds.$$

Uzluksiz $F$ da bu integral tenglama Koshi masalasiga ekvivalentdir.

Boshlang'ich yaqinlashishni $u^{[0]}(t) = a$ deb belgilab, ketma-ketlikni

$$u^{[m+1]}(t) = a + \int_{t_0}^{t} F\left(s, u^{[m]}(s)\right) ds, \quad m \ge 0)$$

formula orqali aniqlaymiz. Bu formula iteratsiyani belgilaydi, yaqinlashuvni esa isbotlash kerak. Agar integral operator yopiq sharni o'ziga akslantirsa va

$q = Lh < 1$ bo'lsa, u qisqartiruvchidir va


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$$\left\|u-u^{[m]}\right\|_C \le \frac{q^m}{1-q}\left\|u^{[1]}-u^{[0]}\right\|_C.$$

Bu erda $\|u\|_C = \max_{t\in[t_0,t_0+h]}\|u(t)\|_\infty$. Baholash Banax prinsipining standart natijasidir [5-15].

$\tau = t - t_0$ deb belgilaymiz. Qidirilayotgan yechimni

$$u_i(t_0+\tau) = a_i + \sum_{n=1}^{\infty} c_{i,n}\,\tau^n, \quad i = 1,2,3$$

ko‘rinishda izlaymiz. $c_{i,n}$ koeffitsientlari $n$-tartibgacha bo‘lgan ma’lumotlardan aniqlanishi lozim. Agar $F_i$ funksiyasi $(t_0, a)$ atrofida analitik bo‘lsa, uning lokal yoyilmasi

$$F_i(t_0+\tau, a+v) = \sum_{p\ge 0}\sum_{\alpha\in\mathbb{N}_0^3} f_{i,p,\alpha}\,\tau^p v_1^{\alpha_1} v_2^{\alpha_2} v_3^{\alpha_3}$$

shakldadir. Bunda $f_{i,p,\alpha}$ tegishli xususiy hosilalarning faktoriallarga bo‘lingan qiymatlaridir. Ikki qatorni tenglamaga qo‘yish va $\tau^n$ oldidagi koeffitsientlarni solishtirish natijasida umumiy rekurrent sxema olinadi:

$$(n+1)c_{i,n+1} = [\tau^n]\,F_i\left(t_0+\tau,\; a+\sum_{k=1}^{n} c_k\,\tau^k\right), \quad n \ge 0.$$

Bu erda $c_k = \left(c_{1,k}, c_{2,k}, c_{3,k}\right)^{\mathsf{T}}$, $[\tau^n]$ - mos koeffitsientni ajratish amali.

$n = 0$ da $c_{i,1} = F_i(t_0, a)$. O‘ng tomon faqat avvalgi koeffitsientlarga bog‘liq.

Hisoblashni aniqlashtirish uchun birinchi ikki koeffitsientni yozamiz.

$J = D_u F(t_0, a)$ - YAkobi matritsasi, $F_t = \partial F / \partial t$ bo‘lsin. U holda

$$c_1 = F(t_0, a), \qquad c_2 = \frac{1}{2}\left(F_t(t_0, a) + Jc_1\right).$$

Uchinchi koeffitsient esa

$$3c_3 = Jc_2 + \frac{1}{2}F_{tt} + F_{tu}c_1 + \frac{1}{2}D_u^2 F[c_1, c_1],$$

bilan aniqlanadi; barcha hosilalar $(t_0, a)$ da olingan. Rekursiyani avtomatlashtirish mumkin.

Quyidagi teorema o‘rinli.

1-teorema. *F funksiyasining har bir komponenti kompleks sohadagi*

$$|t - t_0| \le r, \qquad \| u - a \|_\infty \le \rho$$

*biror atrofida golomorf bo‘lsin va unda* $\| F(t,u) \|_\infty \le M$ *(*$M > 0$*) bajarilsin. U holda Koshi masalasining* $|t - t_0| < h$ *da yagona analitik yechimi mavjud, bunda*

$$h = min\left\{\frac{r}{2}, \frac{\rho}{8M}\right\}.$$

*Ushbu yechimning (vaqt bo‘yicha)* $N$*-tartibli Teylor ko‘phadi uchun* $|t - t_0| < h$ *da*


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$$\left\| u(t) - a - \sum_{n=1}^{N} c_n (t - t_0)^n \right\|_{\infty} \le \frac{\rho}{2} \frac{\theta^{N+1}}{1-\theta}, \quad \theta = \frac{|t - t_0|}{h}$$

*baholash o‘rinli.*

Isbot. $|\tau| \le h$ da golomorf va $\max_{|\tau|\le h} \| u - a \|_{\infty} \le \rho/2$ shartni bajaruvchi funksiyalarning yopiq sharni olamiz. Unda

$$\mathcal{T}u(t) = a + \int_{t_0}^{t} F\left(s, u(s)\right) ds$$

operatorini qaraymiz. Integral kompleks kesma bo‘yicha olinadi.

$\|\mathcal{T}u - a\|_{\infty} \le Mh \le \rho/8$ bo‘lgani uchun shar o‘z-o‘ziga akslanadi. Koshi bahosidan ichki $\rho/2$ sharda har bir $\partial F_i / \partial u_j$ hosilaning moduli $2M/\rho$ dan oshmaydi. Uch komponent mavjudligini hisobga olsak,

$$\|F(t,u) - F(t,v)\|_{\infty} \le \frac{6M}{\rho} \|u - v\|_{\infty}.$$

Demak, $\mathcal{T}$ uchun qisqarish koeffitsienti $6Mh/\rho \le 3/4 < 1$. Banax teoremasi yagona qo‘zg‘almas nuqtani beradi. Pikar iteratsiyalari golomorf bo‘lib, tekis yaqinlashgani sababli ularning limiti ham $|\tau| < h$ da golomorfdir. Koshining integral formulasiga ko‘ra $\|c_n\|_{\infty} \le (\rho/2)h^{-n}$ $(n \ge 1)$. Geometrik qator qoldig‘ini yig‘indilab, teoremadagi bahoni olamiz. Teorema isbotlandi.

Izoh. $h -$ kafolatlangan quyi chegara, aniq radius emas; teorema global vaqt yaqinlashuvini bildirmaydi.

Kichik parametr bo‘yicha konstruktiv yaqinlashish. Umumiy nochiziqli sistema qo‘shimcha ta’sir parametriga ega bo‘lsin:

$$\dot{u} = F_0(t,u) + \varepsilon G(t,u), \qquad u(t_0) = a.$$

Bunda $\varepsilon$ kichik o‘lchamsiz parametr, $F_0$ esa asosiy dinamikani bildiradi. Yechimni $u = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \cdots$ ko‘rinishda qidirib, bir xil darajalar oldidagi hadlarni tenglashtiramiz. Bu erda $u_k(t_0) = 0$ $(k \ge 1)$, $u_0(t_0) = a$.

$$\dot{u}_0 = F_0(t,u_0), \qquad \dot{u}_1 = J_0(t)u_1 + G(t,u_0),$$

$$\dot{u}_2 = J_0(t)u_2 + \frac{1}{2} D_u^2 F_0(t,u_0)[u_1,u_1] + D_u G(t,u_0)u_1,$$

bunda $J_0(t) = D_u F_0\big(t, u_0(t)\big)$. Ikkinchi tenglama ma’lum $u_0$ topilgandan keyin chiziqli, uchinchisi ham $u_2$ ga nisbatan chiziqlidir. Agar $\Phi(t,s)$ - $\dot{v} = J_0(t)v$ ning o‘tish matritsasi bo‘lsa,

$$u_1(t) = \int_{t_0}^{t} \Phi\,(t,s) G\big(s, u_0(s)\big)\, ds.$$

Quyidagi o‘zaro bog‘langan, biroq Volterra shakliga ega bo‘lmagan sistemani qaraymiz:


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$$\dot{x} = -x + \varepsilon yz, \qquad \dot{y} = -2y + \varepsilon xz, \qquad \dot{z} = -4z + \varepsilon xy,$$
$$x(0) = y(0) = z(0) = 1.$$

$\varepsilon = 0$ da nolinchi tartib yechimlari

$$x_0(t) = e^{-t}, \qquad y_0(t) = e^{-2t}, \qquad z_0(t) = e^{-4t}$$

bo‘ladi. Parametr bo‘yicha qatorlarni

$$x = \sum_{n\geq 0} \varepsilon^n x_n, \qquad y = \sum_{n\geq 0} \varepsilon^n y_n, \qquad z = \sum_{n\geq 0} \varepsilon^n z_n$$

deb olamiz. Bir xil $\varepsilon^n$ darajalarini tenglashtirsak, $n \geq 1$ uchun

$$\begin{cases} \dot{x}_n + x_n = \sum_{j=0}^{n-1} y_j z_{n-1-j}, \\ \dot{y}_n + 2y_n = \sum_{j=0}^{n-1} x_j z_{n-1-j}, \\ \dot{z}_n + 4z_n = \sum_{j=0}^{n-1} x_j y_{n-1-j} \end{cases}$$

va $x_n(0) = y_n(0) = z_n(0) = 0$ kelib chiqadi. Integrallovchi ko‘paytuvchi yordamida birinchi tartibda

$$\begin{cases} x_1(t) = \frac{1}{5} e^{-t}(1 - e^{-5t}), \\ y_1(t) = \frac{1}{3} e^{-2t}(1 - e^{-3t}), \\ z_1(t) = e^{-4t}(e^t - 1) \end{cases}$$

natijani olamiz. Masalan, $(e^t x_1)' = e^{-5t}$ tengligini integrallash birinchi formulani beradi.

Ikkinchi tartibli tuzatishlar uchun o‘ng tomonlar $y_1 z_0 + y_0 z_1$, $x_1 z_0 + x_0 z_1$ va $x_1 y_0 + x_0 y_1$ bo‘ladi. Hisoblashdan keyin

$$\begin{cases} x_2(t) = e^{-t}\left[\frac{1 - e^{-4t}}{4} - \frac{2(1 - e^{-5t})}{15} - \frac{1 - e^{-8t}}{24}\right], \\ y_2(t) = e^{-2t}\left[\frac{1 - e^{-2t}}{2} - \frac{4(1 - e^{-3t})}{15} - \frac{1 - e^{-8t}}{40}\right], \\ z_2(t) = e^{-4t}\left[\frac{8}{15}(e^t - 1) - \frac{1 - e^{-2t}}{6} - \frac{1 - e^{-4t}}{20}\right] \end{cases}$$

formulalar kelib chiqadi. Masalan, $z_2$ ni tekshirishda $(e^{4t} z_2)' = \frac{8}{15} e^t - \frac{1}{3} e^{-2t} - \frac{1}{5} e^{-4t}$ tengligi olinadi; integrallash yuqoridagi ifodani beradi. Har uchala $u_1, u_2$ tuzatish $t = 0$ da nolga teng. SHuningdek, ularni tegishli chiziqli rekurrent tenglamalarga bevosita qo‘yish orqali ayniyatlar tekshirildi.


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Ikkinchi tartibli analitik yaqinlashish

$$u^{\langle 2\rangle}(t,\varepsilon) = u_0(t) + \varepsilon u_1(t) + \varepsilon^2 u_2(t)$$

ko‘rinishidadir, $u_j = \left(x_j, y_j, z_j\right)^{\mathsf{T}}$.

2-teorema. $\sum_{n\geq 0} \varepsilon^n\, u_n(t)$ *parametrik qator* $|\varepsilon| < 1/4$ *bo‘lganda manfiy bo‘lmagan barcha vaqtlarda absolyut va tekis yaqinlashadi. Uning yig‘indisi berilgan Koshi masalasining yagona chegaralangan yechimidir. N-tartibli qisman yig‘indi uchun*

$$\left\| u(t,\varepsilon) - \sum_{n=0}^{N} \varepsilon^n\, u_n(t) \right\|_B \leq \frac{(4|\varepsilon|)^{N+1}}{1-4|\varepsilon|}$$

*baholash o‘rinli.*

Isbot. $E(t) = \mathrm{diag}(e^{-t}, e^{-2t}, e^{-4t})$ va $B(v,w) = (v_2 w_3, v_1 w_3, v_1 w_2)^{\mathsf{T}}$ belgilashlarini kiritamiz. $\| E(t) \|_\infty \leq e^{-t}$ hamda $\| B(v,w) \|_\infty \leq \| v \|_\infty \| w \|_\infty$. Integral tenglama

$$u(t) = E(t)1 + \varepsilon \int_0^t E\,(t-s) B\big(u(s), u(s)\big)\, ds$$

ko‘rinishda. CHegaralangan uzluksiz kompleks-vektor funksiyalar fazosida $\| v \|_B$ deb $t \geq 0$ bo‘yicha $\|v(t)\|_\infty$ qiymatlarining eng kichik yuqori chegarasini belgilaymiz. $n$-tartib koeffitsienti

$$u_n(t) = \int_0^t E\,(t-s) \sum_{j=0}^{n-1} B\left(u_j(s), u_{n-1-j}(s)\right)\, ds$$

bilan aniqlanadi. $A_n = \| u_n \|_B$ desak, $A_0 = 1$, $\int_0^t e^{-(t-s)}\, ds \leq 1$ dan

$$A_n \leq \sum_{j=0}^{n-1} A_j\, A_{n-1-j}, \quad n \geq 1$$

kelib chiqadi. Katalan sonlari $C_0 = 1$, $C_n = \sum_{j=0}^{n-1} C_j\, C_{n-1-j}$ rekursiyasini bajaradi. Induksiyaga ko‘ra $A_n \leq C_n \leq 4^n$. SHuning uchun

$$\sum_{n\geq 0} |\varepsilon|^n \| u_n \|_B \leq \sum_{n\geq 0} (4|\varepsilon|)^n < \infty.$$

Qator manfiy bo‘lmagan barcha vaqtlarda tekis yaqinlashganidan integral tenglamada limitga o‘tish mumkin. Hosil bo‘lgan funksiya asl sistemani qanoatlantiradi. YAgonalik uchun integral operatorni $\|u\|_B \leq 2$ sharda qaraymiz: u sharni $|\varepsilon| \leq 1/4$ da o‘ziga akslantirib, $|\varepsilon| < 1/4$ da Lipshits koeffitsienti $4|\varepsilon| < 1$ bo‘lgan qisqartirishdir. Demak, chegaralangan yechim yagona. Katalan bahosidan qolgan hadlarni geometrik qator bilan yig‘ish teoremadagi xatolik formulasini beradi. Isbot tugadi.


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Natija. $|\varepsilon| = 0.1$ va $N = 2$ tanlansa, teorema bo‘yicha barcha $t \geq 0$ uchun xatolik $0.106667$ dan katta emas. Bu qo‘pol, ammo vaqtdan mustaqil kafolat. Qoldiq bahosini kichikroq qilish uchun $\varepsilon$ ni kamaytirish yoki qatorning yuqoriroq tartiblarini hisoblash kifoya; masalan, $|\varepsilon| = 0.02$, $N = 2$ da u $0.000557$ dan kichik bo‘ladi.

Misolning to‘g‘riligini yana bir usul bilan ko‘ramiz. Vaqt bo‘yicha $x = \sum_{n\geq 0} a_n\, t^n$, $y = \sum b_n t^n$, $z = \sum d_n t^n$ desak, $a_0 = b_0 = d_0 = 1$ va

$$\begin{cases} (n+1)a_{n+1} = -a_n + \varepsilon \sum_{k=0}^{n} b_k\, d_{n-k}, \\ (n+1)b_{n+1} = -2b_n + \varepsilon \sum_{k=0}^{n} a_k\, d_{n-k}, \\ (n+1)d_{n+1} = -4d_n + \varepsilon \sum_{k=0}^{n} a_k\, b_{n-k} \end{cases}$$

tengliklar kelib chiqadi. Birinchi ikki bosqichda

$$a_1 = -1 + \varepsilon, \quad a_2 = (1 - 7\varepsilon + 2\varepsilon^2)/2,$$
$$b_1 = -2 + \varepsilon, \quad b_2 = (4 - 7\varepsilon + 2\varepsilon^2)/2,$$
$$d_1 = -4 + \varepsilon, \quad d_2 = (16 - 7\varepsilon + 2\varepsilon^2)/2$$

hosil bo‘ladi. 7-bo‘limdagi $u_0 + \varepsilon u_1 + \varepsilon^2 u_2$ ifodalarini $t = 0$ atrofida yoyganda ushbu koeffitsientlarning $\varepsilon^0, \varepsilon^1, \varepsilon^2$ qismlari aynan takrorlanadi. Bu - vaqt va parametr qatorlari o‘rtasidagi algebraik muvofiqlik tekshiruvidir.

Uch komponentli Lotka-Volterra tipidagi sistemaning umumiy shakli [16-25]

$$\dot{u}_i = u_i \left( r_i + \sum_{j=1}^{3} \beta_{ij}\, u_j \right), \quad i = 1,2,3$$

bo‘ladi. Bu umumiy sistemaning bir xususiy kvadratik holidir. Unda har bir tenglamada $u_i$ ko‘paytuvchi sifatida qatnashgani uchun yagonalik sharti bajarilganda $u_i = 0$ koordinata tekisligi invariantdir. Avvalgi misolda esa bunday tuzilish yo‘q edi. SHunday qilib, Volterra sistemasining ayrim dinamik xususiyatlari umumiy nochiziqli sistema haqida qo‘shimcha shartsiz aytib bo‘lmaydigan xossalardir.

Vaqt bo‘yicha qator $u_i(t) = \sum_{n\geq 0} c_{i,n}\, t^n$ $(c_{i,0} = a_i)$ deb olinsa, umumiy rekursiya bu erda aniq ko‘rinishga keladi:

$$(n+1)c_{i,n+1} = r_i c_{i,n} + \sum_{j=1}^{3} \beta_{ij} \sum_{k=0}^{n} c_{i,k}\, c_{j,n-k}.$$

Misol sifatida siklik ta’sirni ifodalovchi

$$\begin{cases} \dot{x} = x(-1 + \varepsilon y), \\ \dot{y} = y(-2 + \varepsilon z), \\ \dot{z} = z(-4 + \varepsilon x) \end{cases}$$


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sistemani va

$$x(0) = A, y(0) = B, z(0) = C$$

shartlarni olamiz. Nolinchi yaqinlashish $x_0 = Ae^{-t}$, $y_0 = Be^{-2t}$, $z_0 = Ce^{-4t}$. Birinchi tartibli tuzatishlar quyidagi aniq formulalar bilan beriladi:

$$\begin{cases} x_1(t) = \dfrac{AB}{2}e^{-t}(1 - e^{-2t}), \\ y_1(t) = \dfrac{BC}{4}e^{-2t}(1 - e^{-4t}), \\ z_1(t) = ACe^{-4t}(1 - e^{-t}). \end{cases}$$

Har bir formula boshlang‘ich shartni qanoatlantiradi va mos ravishda $x_1' + x_1 = x_0y_0$, $y_1' + 2y_1 = y_0z_0$, $z_1' + 4z_1 = z_0x_0$ tenglamalardan chiqadi. Demak, avvalgi rekurrent va perturbatsion usullar Volterra sinfiga ham qo‘llanadi, ammo bu erdagi birinchi tartibli hisoblashlardan barcha parametrlar uchun global yaqinlashuv haqida xulosa chiqarilmaydi. Bunday xulosa uchun alohida aprior baholar kerak.

Xulosa. Uch noma’lumli umumiy nochiziqli Koshi masalasining analitik yechimlarini qurish uchun ikki bir-birini to‘ldiruvchi konstruktiv sxema ishlab chiqildi: vaqt bo‘yicha Teylor koeffitsientlarini aniqlaydigan umumiy rekurrent munosabat va kichik parametr bo‘yicha ketma-ket chiziqli tuzatishlar. Birinchi sxema kvadratik bo‘lmagan analitik funksiyalarga ham tatbiq etiladi.

Kompleks polidiskdagi cheklanganlik sharti orqali vaqt qatori uchun ochiq lokal radius va qoldiq had bahosi asoslandi. Maxsus uch komponentli kvadratik sistema misolida dastlabki ikki parametrik tuzatish aniq hisoblandi va majoranta qator yordamida $|\varepsilon| < 1/4$ bo‘lganda qatorning manfiy bo‘lmagan barcha vaqtlarda tekis yaqinlashuvi isbotlandi. Bu shart maksimal diapazon deb da’vo qilinmaydi.

Volterra tipidagi sistema uchun alohida rekurrent formula va birinchi parametrik yaqinlashish keltirildi. Umumiy natijalarni ushbu xususiy sinfga tatbiq etish bilan birga, koordinata gipertekisliklarining invariantligi kabi maxsus xossalarning kelib chiqishi ko‘rsatildi. Keyingi tadqiqotlarda majoranta baholarini keskinlashtirish, muvozanat holatlarini hisobga olgan vaznli normalardan foydalanish va uzoq vaqt uchun aniqroq asimptotik yoyilmalarni qurish mumkin.

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