Matematik induksiya usuli orqali o‘quvchilarning isbotlash kompetensiyasini rivojlantirish

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Annotatsiya: Mazkur maqolada matematik induksiya metodi orqali o‘quvchilarning isbotlash kompetensiyasini rivojlantirishning nazariy-didaktik asoslari va metodik imkoniyatlari yoritiladi. Matematik induksiyaning o‘quvchilarda mantiqiy fikrlash, matematik mulohazalarni asoslash, umumlashtirish va mustaqil xulosa chiqarish ko‘nikmalarini shakllantirishdagi ahamiyati tahlil qilinadi. Induksiya metodini o‘qitishda boshlang‘ich holatni tekshirish, induktiv farazni shakllantirish va induktiv o‘tishni asoslash bosqichlariga tayangan metodik yondashuvlar tavsiflanadi. Shuningdek, muammoli topshiriqlar va bosqichma-bosqich murakkablashuvchi masalalar orqali o‘quvchilarning mustaqil isbotlash faoliyatini tashkil etish usullari taklif etiladi. Mazkur yondashuvlarning matematik tafakkurni rivojlantirish va isbotlash kompetensiyasini takomillashtirishdagi didaktik ahamiyati asoslanadi.

Kalit so‘zlar: matematik induksiya, isbotlash kompetensiyasi, mantiqiy fikrlash, induktiv faraz, induktiv o‘tish, matematik tafakkur, muammoli topshiriqlar, o‘qitish metodikasi

Developing students’ proof competence through the method of mathematical induction

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Abstract: This article examines the theoretical and didactic foundations and methodological possibilities of developing students’ mathematical proof competence through the method of mathematical induction. The significance of mathematical induction in fostering students’ logical thinking, ability to justify mathematical statements, generalization skills, and independent conclusion-drawing is analyzed. Methodological approaches based on the stages of verifying the base case, formulating the inductive hypothesis, and justifying the inductive step in teaching

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mathematical induction are described. In addition, methods for organizing students’ independent proof activities through problem-based tasks and progressively more complex mathematical problems are proposed. The didactic significance of these approaches in developing mathematical thinking and improving mathematical proof competence is substantiated.

Keywords: mathematical induction, mathematical proof competence, logical thinking, inductive hypothesis, inductive step, mathematical thinking, problem-based tasks, teaching methodology

Fizika, kimyo, biologiya kabi fanlarda kuzatish va tajribalarga suyanib, induktiv mulohazalar yuritish keng qo‘llaniladi. Induksiya so‘zi o‘zbek tilida “yetaklab borish” yoki “boshqarib borish” kabi ma’nolarni anglatadi.

Xususiy xulosalardan foydalanib umumiy xulosalar chiqarish *induksiya* deyiladi. Matematik induksiya usuli asosida matematik induksiya prinsipi yotadi. Bu prinsipning mazmunini izohlaylik. Biror n natural songa bog‘liq fikrimizni $A(n)$ orqali belgilaylik. Bu mulohazamizning ixtiyoriy n natural son uchun to‘g‘riligini isbotlash talab etilgan bo‘lsin. Lekin $A(n)$ mulohazaning to‘g‘riligini barcha n lar uchun bevosita tekshirib ko‘rishning iloji bo‘lmasin. U holda, $A(n)$ mulohaza matematik induksiya prinsipiga ko‘ra quyidagicha isbotlanadi: $A(n)$ mulohazaning to‘g‘riligi, avvalo,

I. $n=1$ hol uchun tekshiriladi.

II. $n=k$ uchun to‘g‘ri deb faraz qilib, uning to‘g‘riligi $n=k+1$ holat uchun isbotlanadi.

Shundan so‘ng $A(n)$ mulohazamiz barcha n natural sonlar uchun isbotlangan hisoblanadi. Shunday qilib, matematik induksiya prinsipiga asoslangan isbot matematik induksiya usuli bilan isbotlash deyiladi.

A (1) tasdiqning isbotlanishi induksiyaning birinchi qadami hisoblanadi, $A(n)$ uchun farazdan A $(n+1)$ ning isbotlanishi induksiyali o‘tish deyiladi. Bunda induksiya parametri deyiladi, $A(n+1)$ ni isbotlashda $A(n)$ ni faraz qilish induktivli faraz deyiladi.

Matematik induksiya metodining mohiyati quyidagicha:

Agar tasdiqlash ketma-ketligi mavjud bo‘lsa, birinchi tasdiq to‘g‘ri va har bir to‘g‘ri tasdiqdan so‘ng to‘g‘ri tasdiq mavjud bo‘lsa, ketma-ketlikdagi barcha tasdiq to‘g‘ri hisoblanadi.

Shunday qilib, matematik induksiya metodi yordamida isbotlash ikkita qadamdan iborat.

1-qadam . $n = 1$ uchun tasdiq to‘g‘riligi tekshiriladi.

2-qadam. Ixtiyoriy $n=k$ uchun tasdiq to‘g‘ri deb faraz qilinib, navbatdagi $n=k+1$ natural son uchun tasdiq o‘rinli ekanligini isbotlanadi.

Agar ikkala ushbu teoremalar isbotlangan bo‘lsa, matematik induksiya tamoyiliga asoslangan holda, tasdiq ixtiyoriy *n* natural son uchun to‘g‘ri deb xulosa qilinadi.

Eslatma. Barcha natural sonlar uchun emas, balki *n* dan katta yoki teng *m* natural sonlar uchun induksiya bo‘yicha tasdiqni isbotlash zarur bo‘ladi. Bunday holda isbotlash quyidagicha bajariladi.

1-qadam. $n = m$ da tasdiq to‘g‘riligi tekshiriladi.

2-qadam. $n=k$ da tasdiq to‘g‘ri deb faraz qilinib($k \geq m$), $n = k+1$ da tasdiq o‘rinli ekanligi isbotlanadi.

Biz quyida matematik induksiya usuli bilan yaqinroq tanishish maqsadida bir nechta misollar ko‘ramiz.

1-masala. Tenglikni isbotlang: $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$

1-qadam. n=1 da $1 = \frac{1 \cdot (1+1)}{2} \Rightarrow 1 = 1$

2-qadam. n=k da $1 + 2 + 3 + ... + k = \frac{k(k+1)}{2}$ tenglik to‘g‘ri deb faraz qilib, n=k+1 da $1 + 2 + 3 + ... + k + (k+1) = \frac{(k+1)(k+2)}{2}$ tenglikning bajarilishini isbotlaymiz.

$$1 + 2 + 3 + ... + k + (k+1) = \frac{k(k+1)}{2} + (k+1) = \frac{k(k+1) + 2(k+1)}{2} = \frac{(k+1)(k+2)}{2}$$ tenglik isbotlandi.

2-masala. Tenglikni isbotlang. $\frac{1}{2 \cdot 7} + \frac{1}{7 \cdot 12} + ... + \frac{1}{(5n-3)(5n+2)} = \frac{n}{2(5n+2)}$

1-qadam. n=1 da $\frac{1}{2 \cdot 7} = \frac{1}{2(5+2)} \Rightarrow \frac{1}{2 \cdot 7} = \frac{1}{2 \cdot 7}$. 1-qadam isbotlandi.

2-qadam. n=k da $\frac{1}{2 \cdot 7} + \frac{1}{7 \cdot 12} + ... + \frac{1}{(5k-3)(5k+2)} = \frac{k}{2(5k+2)}$ tenglik bajariladi deb n=k+1 holat uchun $\frac{1}{2 \cdot 7} + \frac{1}{7 \cdot 12} + ... + \frac{1}{(5k-3)(5k+2)} + \frac{1}{(5k+2)(5k+7)} = \frac{k+1}{2(5k+7)}$ tenglikni bajarilishini isbotlaymiz. Haqiqatdan ham,

$$\frac{1}{2 \cdot 7} + \frac{1}{7 \cdot 12} + ... + \frac{1}{(5k-3)(5k+2)} + \frac{1}{(5k+2)(5k+7)} = \frac{k}{2(5k+2)} + \frac{1}{(5k+2)(5k+7)} =$$

$$= \frac{5k^2 + 7k + 2}{2(5k+2)(5k+7)} = \frac{(5k+2)(k+1)}{2(5k+2)(5k+7)} = \frac{k+1}{2(5k+7)}$$

Tasdiq isbotlandi.

3-masala. Barcha natural n larda $4^n + 15n - 1$ ifodani 9 ga bo‘linishini isbotlang.

1-qadam. n=1 da $4 + 15 - 1 = 18 \vdots 9$. 1-qadam isbotlandi.

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2-qadam. n=k da $4^k + 15k - 1 \vdots 9$ bajariladi deb, n=k+1 uchun isbotlaymiz: $4^{n+1} + 15(k+1) - 1 = 4 \cdot 4^k + 15k + 15 - 1 = (4^k + 15k - 1) + 3 \cdot (4^k + 5) = (4^k + 15k - 1) + 3 \cdot$ $\cdot ((3+1)^k + 5) = (4^k + 15k - 1) + 3(3A + 1 + 5) = (4^k + 15k - 1) + 3 \cdot (3A + 6) \vdots 9$

Oxirgi holatdan berilgan ifodani 9 ga bo‘linishi isbotlandi.

4-masala. Ixtiyoriy nomanfiy butun n sonlarda $3^{3n+2} + 2^{4n+1}$ ifodani 11 ga bo‘linishini isbotlang.

1-qadam. n=1 da $243 + 32 = 275 \vdots 11$. 1-qadam isbotlandi.

2-qadam. n=k da $3^{3k+2} + 2^{4k+1} \vdots 11$ deb olib, n=k+1 uchun $3^{3(k+1)+2} + 2^{4(k+1)+1} \vdots 11$ ekanligini isbotlaymiz.

$$3^{3(k+1)+2} + 2^{4(k+1)+1} = 3^{3k+2+3} + 2^{4k+1+4} = 27 \cdot 3^{3k+2} + 16 \cdot 2^{4k+1} = 16\left(3^{3k+2} + 2^{4k+1}\right) + 11 \cdot 3^{3k+2} = 11A \vdots 11$$

Tasdiq isbotlandi.

5-masala. Ixtiyoriy $n \in N$ da $n^7 - n$ ifodani 7 ga bo‘linishini isbotlang.

1-qadam. n=1 da $1^7 - 1 = 0 \vdots 7$. 1-qadam isbotlandi.

2-qadam. n=k da $k^7 - k \vdots 7$ bajariladi deb olib, n=k+1 holat uchun isbotlaymiz.

$(k+1)^7 - (k+1) = k^7 + 7k^6 + ... + 7k + 1 - k - 1 = k^7 - k + 7A = 7B \vdots 7$ tenglik isbotlandi.

6-masala. Ixtiyoriy $n \in N$ da $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + n \cdot (n+1) = \frac{n(n+1)(n+2)}{3}$ ayniyatni isbotlang.

1-qadam. n=1 da $1 \cdot 2 = \frac{1 \cdot 2 \cdot 3}{3} \Rightarrow 2 = 2.$ 1-qadam isbotlandi.

2-qadam. n=k da $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + k \cdot (k+1) = \frac{k(k+1)(k+2)}{3}$ to‘g‘ri deb olib n=k+1 holat uchun $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + k \cdot (k+1) + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}$ tenglikni bajarilishini isbotlaymiz.

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + k \cdot (k+1) + (k+1)(k+2) = \frac{k(k+1)(k+2)}{3} + (k+1)(k+2) =$$

$$= \frac{k(k+1)(k+2) + 3(k+1)(k+2)}{3} = \frac{(k+1)(k+2)(k+2)}{3}$$

Tasdiq isbotlandi.

Matematik induksiya metodining tengsizliklarni isbotlashda qo‘llanilishi.

7-masala. Agar $a \geq -1$ bo‘lsa, u holda $(1+a)^n \geq 1 + na, \ \forall n \in N$ Bernulli tengsizligini isbotlang.

1-qadam. n=1 da $(1+a)^1 = 1+a$. Tenglik bajariladi.

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2-qadam. Faraz qilaylik n=k da $(1+a)^k \geq 1+ka$ tengsizlik bajarilsin. Endi n=k+1 hol uchun $(1+a)^{k+1} \geq 1+(k+1)a$ tengsizlikni isbotlaymiz. Haqiqatdan, $(1+a) \geq 0$ bo‘lganda

$$(1+a)^{k+1} = (1+a)^k (1+a) \geq (1+ka)(1+a) = 1+(k+1)a + \underbrace{ka^2}_{\geq 0} \geq 1+(k+1)a$$

bajariladi. Demak, berilgan tengsizlik ixtiyoriy natural son uchun bajariladi.

Xulosa qilib aytganda, matematik induksiya metodini o‘qitish o‘quvchilarning mantiqiy fikrlash, matematik mulohazalarni asoslash va mustaqil isbotlash kompetensiyasini rivojlantirishda muhim ahamiyat kasb etadi. Induksiyaning asosiy bosqichlarini izchil o‘rgatish, muammoli topshiriqlar va murakkablik darajasi ortib boruvchi masalalardan foydalanish o‘quvchilarning isbotlash jarayonini ongli o‘zlashtirishiga xizmat qiladi. Taklif etilgan metodik yondashuvlar matematik tafakkurni rivojlantirish, umumiy qonuniyatlarni aniqlash va asosli xulosalar chiqarish ko‘nikmalarini takomillashtirishga yo‘naltirilgan bo‘lib, matematik ta’lim samaradorligini oshirish uchun didaktik imkoniyatlar yaratadi.

Foydalanilgan adabiyotlar

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