Ikki noma'lumli birinchi tur Fredgolm integral tenglamalar sistemasining yechimi haqida

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Annotatsiya: Ushbu maqolada ikki no'malumli birinchi tur Fredgolm integral tenglamalar sistemasining yechish usuli keltirilgan. Bundan tashqari ikki no'malumli birinchi tur Fredgolm integral tenglamalar sistemasining yechimi misol asosida ham tushuntirilgan.

Kalit so'zlar: Fredgolm birinchi tur integral tenglamasi, Fredgolm ikkinchi tur integral tenglamasi, tenglamalar sistemasi, determinant

On the solution of the system of Fredholm integral equations of the first kind with two unknowns

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Abstract: This article presents the solution of the Fredholm first-type integral equation with two non-integers. In addition, the solution of Fredholm first-type integral equation with two non-integers is also explained on an example basis.

Keywords: Fredholm first-type integral equation, Fredholm second-type integral equation, system of equations, determinant

Ushbu ko'rinishdagi integral tenglama Fredgolmning birinchi tur integral tenglamasi deyiladi:

$$\lambda \int_a^b K(x,t)u(t)dt = f(x) \quad (1)$$

Bunda $u(t)$-noma'lum funksiya, $f(x)$ - ozod had va $K(x,t)$ - tenglamaning yadrosi - ma'lum funksiyalar, integrallash chegaralari a va b berilgan haqiqiy o'zgarmas sonlardir [1].

Fredgolmning ikkinchi tur tenglamasi deb ushbu

$$u(x) = f(x) + \lambda \int_a^b K(x,t)u(t)dt \ (2)$$

tenglamaga aytiladi.Bu tenglamadagi no'malum funksiya $u(x)$ integral ishorasidan tashqarida ham ishtirok etmoqda. (1) va (2) tenglamalardagi λ tenglamaning parametri deb ataladi.

Bu tenglamalardagi $f(x)$ funksiya $I(a \le x \le b)$ kesmada, $K(x,t)$ yadro esa $P(a \le x \le b, a \le t \le b)$ yopiq sohada berilgan deb hisoblanadi [2].

$$\begin{cases} \int_a^b K_{11}(x,s)\varphi_1(s)ds + \int_a^b K_{12}(x,s)\varphi_2(s)ds = f_1(x) \\ \int_a^b K_{21}(x,s)\varphi_1(s)ds + \int_a^b K_{22}(x,s)\varphi_2(s)ds = f_2(x), \end{cases} \quad (3)$$

bu yerda $K_{ij}(x,s) \in C([a,b]^2)$, $f_i(x) \in C[a,b]$, $i,j = 1,2$.

(3) tenglamalar sistemasining yechimi deganda $[a,b]$ kesmada uzluksiz bo'lib, berilgan tenglamalar sistemasining har ikkala tengligini qanoatlantiruvchi $y_i(x), i = 1,2$ funksiyalar tushuniladi.

Aytaylik $E = \{e_1(x), e_2(x)\}$ - X to'plamda bazis va

$$W(x) = det\begin{pmatrix} e_1(x) & e_2(x) \\ e_1'(x) & e_2'(x) \end{pmatrix}$$

bo'lsin.

1-lemma: $\{e_1(x), e_2(x)\}$ funksiyalar sistemasi X fazoda bazis bo'lishi uchun har bir $x \in X$ uchun $W(x) \neq 0$ bo'lishi zarur va yetarli [3].

1) $e_1(x) = \sin x, e_2(x) = \cos x$ bo'lsin. $\{e_1(x), e_2(x)\}$ sistema $X = R$ da bazis tashkil qiladi.

Haqiqatan ham

$$W(x) = det\begin{pmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{pmatrix} = -sin^2(x) - cos^2(x) = -1 \neq 0$$

1-lemmaga ko'ra $\{\sin x, \cos x\}$ sistema basis tashkil qiladi.

2) $e_1(x) = x, e_2(x) = x^2 \ va \ x = (0,1)$ bo'lsin.

$$W(x) = det\begin{pmatrix} x & x^2 \\ 1 & 2x \end{pmatrix} = 2x^2 - x^2 = x^2 \neq 0, x \in X.$$

1-lemmaga ko'ra $\{x, x^2\}$ sistema $x = (0,1)$ to'plamda basis tashkil qiladi.

(3) tenglamalar sistemasida berilgan funksiyalar $\{e_1(x), e_2(x)\}$ bazis bo'yicha quyidagi yoyilmaga ega bo'lsin.

$$\begin{gathered} K_{ij}(x,s) = K_{ij}^{(1)}(s)e_1(x) + K_{ij}^{(2)}(s)e_2(x) \\ f_i(x) = f_i^{(1)}e_1(x) + f_i^{(2)}e_2(x), i = 1,2 \end{gathered} \quad (4)$$

Bu yerda $f_i^{(1)}$, $f_i^{(2)}$ - o'zgarmas son, U holda (3) tenglamalar sistemasining birinchi tengligi quyidagi ko'rinishga keladI [4-8]:

$$\int_a^b [K_{11}^1(s)e_1(x) + K_{11}^2(s)e_2(x)]y_1(s) +$$

$$\int_a^b [K_{12}^1(s)e_1(x) + K_{12}^2(s)e_2(x)]y_2(s)ds = f_1^{(1)}e_1(x) + f_1^{(2)}e_2(x)$$

$$\left[\int_a^b K_{11}^1(s)y_1(s)ds + \int_a^b K_{12}^1(s)y_2(s)ds\right]e_1(x) +$$

$$+\left[\int_a^b K_{11}^2(s)y_1(s)ds + \int_a^b K_{12}^2(s)y_2(s)ds\right]e_2(x) = f_1^{(1)}e_1(x) + f_1^{(2)}e_2(x)$$

Bu tenglikdan quyidagi tenglamalarni hosil qilamiz:

$$\begin{cases} \int_a^b K_{11}^1(s)y_1(s)ds + \int_a^b K_{12}^1(s)y_2(s)ds = f_1^{(1)} \\ \int_a^b K_{11}^2(s)y_1(s)ds + \int_a^b K_{12}^2(s)y_2(s)ds = f_1^{(2)} \end{cases} \quad (5)$$

(4) ifodalarni (3) tenglamalar sistemasining ikkinchi tengligiga qo'yib, quyidagi tengliklarni hosil qilamiz:

$$\int_a^b [K_{21}^1(s)e_1(x) + K_{11}^2(s)e_2(x)]y_1(s)ds +$$

$$\int_a^b [K_{22}^1(s)e_1(x) + K_{22}^2(s)e_2(x)]y_2(s)ds = f_2^{(1)}e_1(x) + f_2^{(2)}e_2(x)$$

Yoki

Oxirgi tenglikdan quyidagi tenglamani hosil qilamiz :

$$\begin{cases} \int_a^b K_{21}^1(s)y_1(s)ds + \int_a^b K_{22}^1(s)y_2(s)ds = f_2^{(1)} \\ \int_a^b K_{11}^2(s)y_1(s)ds + \int_a^b K_{22}^2(s)y_2(s)ds = f_2^{(2)} \end{cases} \quad (6)$$

(5) va (6) tenglamalarni umumlashtirib, hosil bo'lgan tenglamalar sistemasining yechimini $\{e_1(x), e_2(x)\}$ bazis yoyilmasi bo'yicha izlaymiz:

$$y_i(x) = y_i^{(1)}e_1(x) + y_i^{(2)}e_2(x), i = 1,2. (7)$$

Bu yerda $y_i^{(1)}$ va $y_i^{(2)}$, $i = 1,2$ no'malum o'zgarmas sonlar.

(7) ni (5) va (6) tenglamalarga qo'yib quyidagi tenglamalar sistemasini hosil qilamiz:

$$\begin{cases} \int_a^b K_{11}^1(s)\left[y_1^{(1)}e_1(s)+y_1^{(2)}e_2(s)\right]ds+\int_a^b K_{12}^1(s)\left[y_2^{(1)}e_1(s)+y_2^{(2)}e_2(s)\right]ds=f_1^1 \\ \int_a^b K_{11}^2(s)\left[y_1^{(1)}e_1(s)+y_1^{(2)}e_2(s)\right]ds+\int_a^b K_{12}^2(s)\left[y_2^{(1)}e_1(s)+y_2^{(2)}e_2(s)\right]ds=f_2^1 \\ \int_a^b K_{21}^1(s)\left[y_1^{(1)}e_1(s)+y_1^{(2)}e_2(s)\right]ds+\int_a^b K_{22}^1(s)\left[y_2^{(1)}e_1(s)+y_2^{(2)}e_2(s)\right]ds=f_2^1 \\ \int_a^b K_{21}^2(s)\left[y_1^{(1)}e_1(s)+y_2^{(2)}e_2(s)\right]ds+\int_a^b K_{22}^2(s)\left[y_2^{(1)}e_1(s)+y_2^{(2)}e_2(s)\right]ds=f_2^2 \end{cases}$$

bu tenglamalar sistemasida

$$a_{ijk}^{(e)}=\int_a^b k_{ik}^{(e)}(s)e_k(s)ds$$

kabi belgilash kiritsak,

$$\begin{cases} a_{111}^{(1)}y_1^{(1)}+a_{112}^{(1)}y_1^{(2)}+a_{121}^{(1)}y_2^{(1)}+a_{122}^{(1)}y_2^{(1)}=f_1^{(1)} \\ a_{111}^{(2)}y_1^{(1)}+a_{112}^{(2)}y_1^{(2)}+a_{121}^{(2)}y_2^{(1)}+a_{122}^{(2)}y_2^{(2)}=f_2^{(1)} \\ a_{211}^{(1)}y_1^{(1)}+a_{212}^{(1)}y_1^{(2)}+a_{221}^{(1)}y_2^{(1)}+a_{222}^{(1)}y_2^{(2)}=f_2^{(1)} \\ a_{211}^{(1)}y_1^{(1)}+a_{212}^{(2)}y_1^{(2)}+a_{221}^{(2)}y_2^{(1)}+a_{222}^{(2)}y_2^{(1)}=f_2^{(2)} \end{cases} \quad 8)$$

tenglamalar sistemasi hosil bo'ladi.

$$\Delta=\det\begin{pmatrix} a_{111}^{(1)} & a_{112}^{(1)} & a_{121}^{(1)} & a_{122}^{(1)} \\ a_{111}^{(2)} & a_{112}^{(2)} & a_{121}^{(2)} & a_{122}^{(2)} \\ a_{211}^{(1)} & a_{212}^{(1)} & a_{221}^{(1)} & a_{222}^{(1)} \\ a_{211}^{(2)} & a_{212}^{(1)} & a_{221}^{(1)} & a_{222}^{(1)} \end{pmatrix} \quad (9)$$

bo'lib, $\Delta_i^{(e)}$ esa (9) determinantda ustun elementlarini $(f_1^{(1)}, f_1^{(2)}, f_2^{(1)}, f_2^{(2)})^T$ bilan almashtirishdan hosil bo'lgan matritsa determinant, ya'ni

$\Delta_1^{(1)}$ da birinchi ustun; $\Delta_1^{(2)}$ da ikkinchi ustun;

$\Delta_2^{(1)}$ da uchinchi ustun; $\Delta_2^{(2)}$ da esa to'rtinchi ustun;

almashtirilgan.

$\Delta \neq 0$ bo'lsa, u holda (1) tenglamalar sistemasi yagona yechimga ega. Bunda

$$y_i^{(e)}=\frac{\Delta_i^{(a)}}{\Delta}, i,e=1,2.$$

Misol.

$$\begin{cases} \int_0^1 (3x^2+2sx)y_1(s)ds + \int_0^1 (2x^2+3sx)y_2(s)ds = x^2+2x \\ \int_0^1 (x^2+3sx)y_1(s)ds + \int_0^1 (3x^2+sx)y_2(s)ds = x^2+4x \end{cases}$$

tenglamalar sistemasini (0,1) da $\{x^2, x\}$ bazis bo‘yicha yechimini toping: yuqorida bir $\{x^2, x\}$ sistema (0,1) da basis tashkil qilishini ko‘rsatgan edik.

$$K_{11}(x,s) = 3x^2 + 2sx;\ K_{12}(x,s) = 2x^2 + 3sx;$$
$$K_{21}(x,s) = x^2 + 3sx;\ K_{22}(x,s) = 3x^2 + sx;$$
$$f_1(x) = x^2 + 2x;\ f_2(x) = x^2 + 4x;$$

Bu tengliklardan ko’rinib turibdiki,

$$K_{11}^{(1)}(s) = 3, K_{11}^{(2)}(s) = 2s, K_{12}^{(1)}(s) = 2, K_{12}^{(2)}(s) = 3s,$$
$$K_{21}^{(1)}(s) = 1, K_{21}^{(2)}(s) = 3s, K_{22}^{(1)}(s) = 3, K_{22}^{(2)}(s) = s,$$
$$f_1^{(1)} = 1, f_1^{(2)} = 2, f_2^{(1)} = 1, f_2^{(2)} = 1.$$

U holda

$$a_{111}^{(1)} = \int_0^1 3x^2dx = 1;\ a_{112}^{(2)} = \int_0^1 2xxdx = \frac{2}{3};$$
$$a_{111}^{(2)} = \int_0^1 2xx^2dx = \frac{1}{2};\ a_{121}^{(2)} = \int_0^1 3xx^2dx = \frac{3}{4};$$
$$a_{112}^{(1)} = \int_0^1 3xdx = \frac{3}{2};\ a_{122}^{(2)} = \int_0^1 3xxdx = 1;$$
$$a_{121}^{(2)} = \int_0^1 2x^2dx = \frac{2}{3};\ a_{211}^{(2)} = \int_0^1 1x^2dx = \frac{1}{3};$$
$$a_{122}^{(1)} = \int_0^1 2xdx = 1;\ a_{212}^{(2)} = \int_0^1 1xdx = \frac{1}{2};$$
$$a_{221}^{(2)} = \int_0^1 3x^2dx = 1;\ a_{121}^{(2)} = \int_0^1 3xxdx = 1;$$
$$a_{222}^{(1)} = \int_0^1 3xdx = \frac{3}{2};\ a_{221}^{(2)} = \int_0^1 xx^2dx = \frac{1}{4};$$
$$a_{112}^{(1)} = \int_0^1 3xx^2dx = \frac{3}{4};\ a_{222}^{(2)} = \int_0^1 xxdx = \frac{1}{3};$$

$$\Delta = \begin{vmatrix} 1 & 3/2 & 2/3 & 1 \\ 1/2 & 2/3 & 3/4 & 1 \\ 1/3 & 1/2 & 1 & 3/2 \\ 3/4 & 1 & 1/4 & 1/3 \end{vmatrix} = -\frac{49}{5184} \neq 0.$$

Demak, berilgan tenglamalar sistemasi $\{x^2, x\}$ bazis bo‘yicha yagona yechimga ega [9-17].

$$\Delta_1^{(1)} = -\frac{49}{108}, \Delta_1^{(2)} = -\frac{259}{864}, \Delta_2^{(1)} = \frac{35}{216}, \Delta_2^{(2)} = -\frac{49}{492},$$
$$y_2^{(1)} = -\frac{120}{7}, y_2^{(2)} = 12.$$

Berilgan tenglamaning yechimi quyidagicha bo'ladi:

$$y_1(x) = 48x^2 - \frac{222}{7}x, y_2(x) = -\frac{120}{7}x^2 + 12x.$$

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