Formulalarni keltirib chiqarish va teoremalarni isbotlashda koordinatalar usulining tatbiqlari

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Annotatsiya: Ushbu maqolada Yevklid geometriyasining ba'zi teorema va masalalarida berilgan shakllarni koordinatalar sistemasiga qulay joylashtirib, isbotlash yoki yechishga asoslangan “Koordinatalar usuli” ning ba’zi formulalarni keltirib chiqarish va teoremalarni isbotlashga tatbiqi haqida bir qancha metodik tavsiyalar berib o‘tilgan.

Kalit so‘zlar: koordinatalar usuli, Yevklid geometriyasi, vektorlar ustida amallar, Ptolomey va Menelay teoremalari, isbot, mediana, bissektrissa, masala, yechim

Applications of the method of coordinates in deriving formulas and proving theorems

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Abstract: In this article, some methodological recommendations are given on the application of the "Method of Coordinates", which is based on the proof or solution of some theorems and problems of Euclidean geometry, by conveniently placing the shapes given in the coordinate system, to derive some formulas and prove theorems.

Keywords: coordinate method, Euclidean geometry, operations on vectors, theorems of Ptolemy and Menelaus, proof, median, bisector, problem, solution

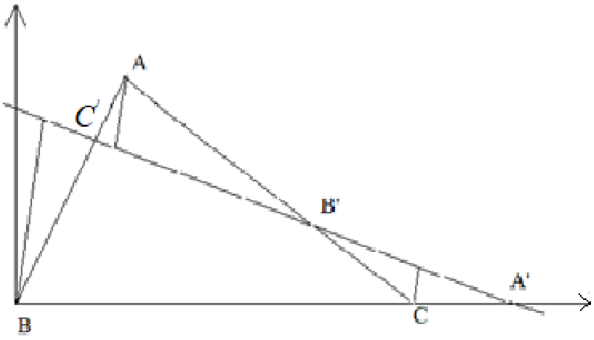
Ma’lumki, Yevklid geometriyasida masalalar yechish, teoremalarni isbotlash hamda formulalarni keltirib chiqarishning ko‘pgina usullari mavjud, yana bir usul “Koordinatalar usuli” hisoblanadi [1, (N.N.Rakhimov, Year:2019)]

Bu usulning mohiyati shundan iborat: bilamizki, shaklni har qanday holatda dekart koordinatalar sistemasiga tasvirlamaylik u o‘zining perimetri, yuzasi, burchak kattaligi, tomonlari uzunliklari kabi parametrlarini o‘zgartirmaydi. Bundan kelib chiqqan holda shaklning bir uchini koordinatalar boshiga yoki istalgan bir tomonini *Ox* yoki *Oy* o‘qlardan biriga qulay joylashtirish yo‘li bilan masalalar yechiladi. Bu usul yordamida bir qancha teoremalarni dekart koordinatalar sistemasida isbotlash mumkin

[2].

Quyida ushbu usuldan foydalangan holda bir nechta teorema isbotlarini hamda formulalarni keltirib chiqarish borasida ba’zi-bir tavsiyalarni berib o‘tamiz.

1-teorema (Menelay teoremasi).



1-shakl

Agar biror to‘g‘ri chiziq ABC uchburchakning ikkita tomonini va uchinchi tomon davomini mos ravishda C', A', B' nuqtalarda kesib o‘tsa, u holda:

$$\frac{BA'}{A'C}\cdot\frac{CB'}{B'A}\cdot\frac{AC'}{C'B}=-1$$ tenglik o‘rinli bo‘ladi [2].

Bu yerda $\alpha,\beta,\gamma-$ uchlarning mos ravishda perpendikularlari.

Isbot. Uchburchakni 1-shakl kabi koordinatalar sistemasiga joylashtirib olamiz. U holda uchburchak uchlarining koordinatalari quyidagicha:

A(x_1,y_1), B(0,0), C(x_4,0), A\`($x_5$,0), B\`(x_3,y_3), C\`(x_2,y_2).

Uchburchaklar o‘xshashligidan quyidagilarni yoza olamiz:

$$\frac{CB`}{B`A}=\frac{\sqrt{(x_3-x_4)^2+{y_3}^2}}{\sqrt{(x_1-x_3)^2+(y_1-y_3)^2}}=\frac{\gamma}{\alpha} \quad (1)$$

$$\frac{BA`}{A`C}=\frac{x_5}{x_5-x_4}=-\frac{\beta}{\gamma} \quad (2)$$

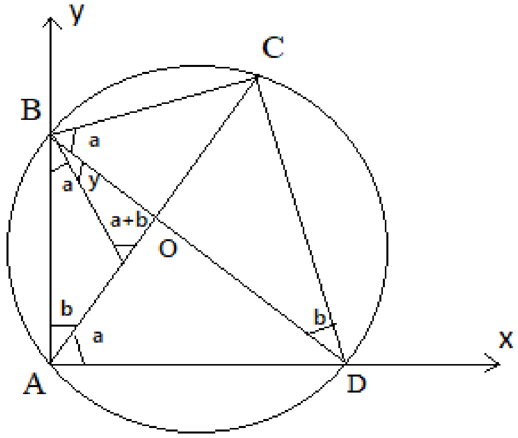
$$\frac{AC`}{C`B}=\frac{\sqrt{(x_2-x_1)^2+(y_2+y_1)^2}}{\sqrt{{x_2}^2+{y_2}^2}}=\frac{\alpha}{\beta} \quad (3)$$

(1), (2) va (3) tengliklarni hadma-had ko‘paytirsak:

$$\frac{x_5}{x_5-x_4}\cdot\frac{\sqrt{(x_3-x_4)^2+{y_3}^2}}{\sqrt{(x_1-x_3)^2+(y_1-y_3)^2}}\cdot\frac{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}{\sqrt{{x_2}^2+{y_2}^2}}=-1$$

natijani olamiz. Teorema isbot bo‘ldi.

2-teorema (Ptolomey teoremasi).



2-shakl

Doiraga ichki chizilgan to‘rtburchak diagonallarining ko‘paytmasi uning qarama-qarshi tomonlar ko‘paytmasining yig‘indisiga teng bo‘ladi, ya’ni $AC\cdot BD = AB\cdot CD + BC\cdot AD$ munosabat o‘rinli bo‘ladi [3].

Isbot. 2-shaklda kelib chiqib:

A(0,0), B(0,y), C(x,y), D(x,0), E(x_2, y_2) larni yozamiz. Uchurchaklarning mos o‘xshashligidan :

ABE~BCD $$\frac{\sqrt{(x-x_1)^2+y_1^2}}{\sqrt{(x_2^2+y_2^2)}} = \frac{\sqrt{x^2+y^2}}{\sqrt{y^2}}$$

$$y\sqrt{(x-x_1)^2+y_1^2} = \sqrt{x^2+y^2}\cdot\sqrt{x_2^2+y_2^2} \quad (4)$$

BEC~ABD $$\frac{\sqrt{x_1^2+(y_1-y)^2}}{\sqrt{x^2+y^2}} = \frac{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}{\sqrt{x^2}}$$

$$x\cdot\sqrt{x_1^2+(y_1-y)^2} = \sqrt{x^2+y^2}\cdot\sqrt{(x_1-y_1)^2+(y_1-y_2)^2} \quad (5)$$

(4) va (5) tengliklarni qo‘shib yuborib:

$$y\cdot\sqrt{(x-x_1)^2+y_1^2} + x\cdot\sqrt{x_1^2+(y_1-y)^2} =$$

$$= \sqrt{x^2+y^2}\cdot\sqrt{x_2^2+y_2^2} + \sqrt{x^2+y^2}\cdot\sqrt{(x_1-x_2)^2+(y_1-y_2)^2} =$$

$$= \sqrt{x^2+y^2}\left(\sqrt{x_2^2+y_2^2} + \sqrt{(x_1-x_2)^2+(y_1-y_1)^2}\right)$$

Qavs ichidagi ifoda 2-shaklga ko‘ra:

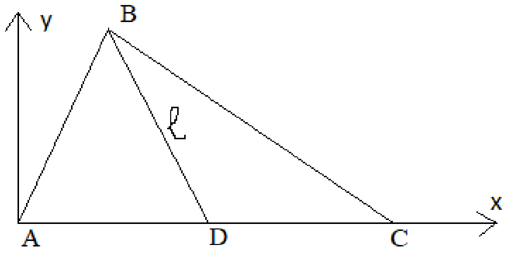
$\sqrt{x^2+y^2}\cdot\sqrt{x_1^2+y_1^2}$ ga teng ekanligi kelib chiqadi.

Demak, tenglik quyidagi ko‘rinishga kelar ekan:

$y\cdot\sqrt{(x-x_1)^2+y_1^2} + x\cdot\sqrt{x_1^2+(y_1-y)^2} = \sqrt{x^2+y^2}\cdot\sqrt{x_1^2+y_1^2}$. Teorema isbotlandi.

Quyida shu usul bilan isbotlangan formulalarni keltiramiz.

1-masala. ABC uchburchakka 3-shakldagidek ℓ - bissektrissa o‘tkazilgan bo‘lsin.



3-shakl

Bunda, $l=\sqrt{AB\cdot BC-AD\cdot DC}$ tenglikning o‘rinli ekanligini isotlang.

Yechim. 3-shakldan A(0,0), B(x_1, y_1), C(x_2,0), D(x,0) lar ma’lum. AD va CD tomonlar uchun kosinuslar teoremasini qo‘llab quyidagilarga ega bo‘lamiz:

$$x^2=\left(\sqrt{x_1^2+y_1^2}\right)^2+\ell^2-2\sqrt{x_1^2+y_1^2}\cdot\ell\cdot\cos\alpha$$

$$(x_2-x)^2=\left(\sqrt{(x_2-x_1)^2+y_1^2}\right)^2+\ell^2-2\sqrt{(x_2-x_1)^2+y_1^2}\cdot\ell\cdot\cos\alpha$$

Bu ikkala formuladan ham ℓ ni topamiz:

$$\ell=\frac{\left(\sqrt{x_1^2+y_1^2}\right)^2+\ell^2-x^2}{2\cdot\cos\alpha\cdot\sqrt{x_1^2+y_1^2}}\quad \ell=\frac{\left(\sqrt{(x_2-x_1)^2+y_1^2}\right)^2+\ell^2-(x_2-x)^2}{2\cdot\cos\alpha\cdot\sqrt{(x_2-x_1)^2+y_1^2}}$$

Hosil bo‘lgan tengliklarni tenglashtirib, soddalashtirsak:

$$\ell^2\cdot\left(\sqrt{x_1^2+y_1^2}-\sqrt{(x_2-x_1)^2+y_1^2}\right)=\sqrt{x_1^2+y_1^2}\sqrt{(x_2-x_1)^2+y_1^2}\,\cdot$$

$$\cdot\left(\sqrt{x_1^2+y_1^2}-\sqrt{(x_2-x_1)^2+y_1^2}\right)-x\cdot(x_2-x)\left(\sqrt{x_1^2+y_1^2}-\sqrt{(x_2-x_1)^2+y_1^2}\right)$$

Ushbu tenglikning ikkala tarafida qavs ichidagi bir xil ko‘phadlarni qisqartirib:

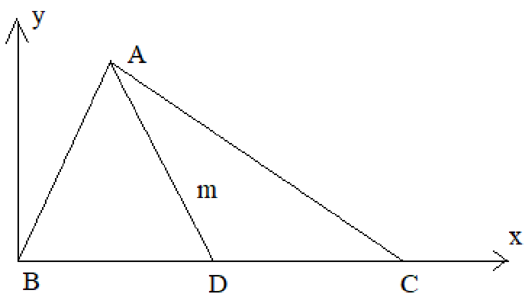
$$\ell^2=\sqrt{\left(x_1^2+y_1^2\right)\left((x_2-x_1)^2+y_1^2\right)}-x\cdot(x_2-x)$$

$$\ell=\sqrt{\sqrt{\left(x_1^2+y_1^2\right)\left((x_2-x_1)^2+y_1^2\right)}-x\cdot(x_2-x)}$$

larga ega bo‘lamiz. Demak, formula isbotlandi.

2-masala. ABC uchburchak 4-shakldagidek berilgan bo‘lsin. U holda:

$$m=\frac{\sqrt{2\left(AC^2+AB^2\right)-BC^2}}{2}$$ formula o‘rinli bo‘ladi.



4-shakl

Yechim. 4-shakldan A(x_1, y_1), B(0,0), C(2x,0), D(x,0), AD=m lar ma’lum. Formulani isbotlash uchun:

$$AD - \frac{\sqrt{2\left(AC^2 + AB^2\right) - BC^2}}{2} = 0$$

ayirmaning nolga tengligini isbotlash kerak bo‘ladi:

$$\sqrt{(x - x_1)^2 + y_1{}^2} - \frac{\sqrt{2 \cdot \left((2x - x_1)^2 + y_1{}^2 + x_1{}^2 + y_1{}^2\right) - 4x^2}}{2} = \sqrt{x^2 - 2xx_1 + x_1{}^2 + y_1{}^2} -$$

$$- \frac{\sqrt{2\left(4x^2 - 4xx_1 + x_1{}^2 + y_1{}^2 + x_1{}^2 + y_1{}^2\right) - 4x^2}}{2} = \sqrt{x^2 - 2xx_1 + x_1{}^2 + y_1{}^2} -$$

$$- \frac{\sqrt{4\left(2x^2 - 2xx_1 + x_1{}^2 + y_1{}^2\right) - 4x^2}}{2} = \sqrt{x^2 - 2xx_1 + x_1{}^2 + y_1{}^2} - \sqrt{x^2 - 2xx_1 + x_1{}^2 + y_1{}^2} = 0$$

formula isobotlandi.

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